Probability Model Based on Zero Truncation of Himanshu Distribution and Their Applications.

Abstract- In this paper we have developed the probability model to study the distribution of number of male migrants from households and pattern of child mortality through zero truncated Himanshu Distribution (ZTHD). The parameter of the proposed model is estimated by method of moment and method of maximum likelihood and its suitability is tested by applying on observed migration data, and collected from various surveys.

Keywords - Migration, Himanshu distribution, Zero truncated distribution, Estimation technique.

1. Introduction

Probability model based on distributions play an important role in the various fields of Social Sciences, Medical Sciences, Environmental Sciences etc.Recently Pandey and Jai kishun(2010), Pandey etal.(2014), Dubey and Pandey(2022) has suggested probability models for vital events.

Zero truncated distributions are certain discrete distributions having support the set of positive integers. These distributions are applicable for the situations when the data to be modelled originate from a mechanism that generates data excluding zero-counts.

Let $p_0(x;\theta)$ is the original distribution with support non-negative positive integers, then the zero truncated version of $p_0(x;\theta)$ with support set of positive integers is given by-

$$p(x;\theta) = \frac{p_0(x;\theta)}{1 - p_0(x;\theta)};\ x = 1,2,3,\dots \qquad (1.1)$$

Agarwal and Pandey (2022) has introduced Himanshu distribution having probability mass function as-

$$P(X = x) = p^n(1 - p^n)^x\ ; \quad \left.\begin{matrix} x = 0,1,2,\dots \\ 0 < p < 1 \\ n \in I^+ \end{matrix}\right\} \qquad (1.2)$$

$$\text{with Mean } = \frac{1 - p^n}{p^n} \text{ and Variance } = \frac{1 - p^n}{p^{2n}}$$

2. Proposed Probability Model -

Using (1.1) and (1.2) the probability mass function of zero truncated Himanshu distribution (ZTHD) given by –

$$P(X = x) = p^n(1 - p^n)^{x-1}\ ; \quad \left.\begin{matrix} x = 1,2,3,\dots \\ 0 < p < 1 \\ n \in I^+ \end{matrix}\right\} \qquad (2.1)$$

The moment generating function of (2.1) is given as-

$$M_X(t) = \sum_{x=1}^{\infty} e^{tx} p^n (1-p^n)^{x-1}$$

$$M_X(t) = \frac{p^n e^t}{1 - e^t(1-p^n)} \quad (2.2)$$

Then the first four moments (about origin) are as follows-

Mean $= \mu_1' = \frac{1}{p^n}$ $\qquad \mu_2' = \frac{2-p^n}{p^{2n}}$

$$\mu_3' = \frac{(p^{2n} - 6p^n + 6)}{p^{3n}} \quad \text{and} \quad \mu_4' = \frac{-(p^{3n} - 14p^{2n} + 36p^n - 24)}{p^{4n}} \quad (2.3)$$

Now the first four (central moments) in given by-

$\mu_1 = 0,$ $\qquad \mu_2 = \frac{1-p^n}{p^{2n}}$

$$\mu_3 = \frac{(1-p^n)(2-p^n)}{p^{3n}} \quad \text{and} \quad \mu_4 = \frac{(1-p^n)(p^{2n} - 9p^n + 9)}{p^{4n}} \quad (2.4)$$

$$\gamma_1 = \frac{2-p^n}{\sqrt{1-p^n}} > 0$$

⇒ Distribution is positivity skewed

$$\gamma_2 = \frac{p^{2n} - 6p^n + 6}{1-p^n}$$

⇒ Distribution is leptokurtic

C.V $= \sqrt{1-p^n}$ and Index of disperion $= \frac{1}{p^n} - 1$

3. Estimation of parameter of ZTHD –

The parameter p of the proposed model (2.1) is estimated by maximum likelihood estimation method in the following way –

$$L = \prod_{i=1}^{k} f(x_i\,; p)$$

$$L = p^{nk}(1-p^n)^{\sum_{i=1}^{k} x_i - k}$$

Now taking log and upon differentiating w.r.t p and equating to zero we get -

$$\Rightarrow \hat{p} = \left(\frac{1}{\bar{x}}\right)^{\frac{1}{n}}$$

Again the parameter p estimated by method of moments after using (2.3) we get –

$$\bar{x} = \mu_1'$$

$$\Rightarrow \hat{p} = \left(\frac{1}{\bar{x}}\right)^{\frac{1}{n}}$$

4. Application

The proposed probability model at n=2 is fitted using some real data sets collected from different sample surveys entitled “Demographic survey of Chandauli district (Rural Area- 2001-2002)”- (2015); Rural development and population growth survey 1978-PRC, BHU” (2015); and “the survey collected by the researcher in Varanasi district (2018) for the single adult male migrant of 15 years and above.

Mortality does not depend only on the biological and epidemiological factors, it depends upon the prevailing health conditions, Medical facilities, environmental conditions and some other socio-economic and cultural factors. In developing and under developing countries the mortality among infants and children is found much higher than that among youngsters. These reasons the high infant mortality has thrown a serious challenge to the doctors, medical personnel’s and is considered as one of the sensitive position of existing medical and health facilities in the population. Therefore it is an attempt to study the infant mortality pattern through our proposed model. In this respect the real data set of Sri Lanka taken from the survey by Meegama (1980) and the real data set of India is taken from Lal (1955).

Table 1. Observed and expected number of household with at least one male migrant according to the number of male migrants aged 15 years and above (survey 2001 data)		
Number of migrants	**Observed no. of households**	**Expected no. of households**
1	97	97.37
2	35	38.11
3	19	14.91
4	6 } 9	9.61
5	3	
Total	**160**	**160**
Mean=1.643 Variance=1.057 $\hat{p}$ =0.7801	χ^2 = 1.41 (after pooling) p-value=0.4941 $\chi^2_{(2)}$ = 5.99 at 5% level of signi icance	

Table 2. Observed and expected number of households with at least one male migrant according to the number of male migrants aged 15 years and above (survey 1978 data)		
Number of migrants	**Observe no. of households**	**Expected no. of households**
1	375	384.32
2	143	133.97
3	49	46.70
4	17	16.28
5+	6	8.72
Total	**590**	**590**
Mean=1.53 Variance=0.8213 $\hat{p}$ =0.8084	χ^2 = 1.82 (after pooling) p-value=0.6105 $\chi^2_{(3)}$ = 7.815 at 5% level of signi icance	

Table 3. Observed and expected number of households with at least one male migrant according to the number of male migrants aged 15 years and above (survey 1978 data) in three types of households.						
Number of migrants	**Type of households**					
	Semi urban		**Remote**		**Growth centre**	
	Observed	**Expected**	**Observed**	**Expected**	**Observed**	**Expected**
1	95	89.4	176	176.4	154	152.3
2	19	27.4	59	58.0	47	51.4
3	10		18	19.0	18	17.3
4	2 } 15	12.2	6 } 10	9.6	9 } 11	8.9
5	3		4		2	
Total	**129**	**129**	**263**	**263**	**230**	**230**
Mean=	1.44		1.49		1.51	
Variance=	0.6374		0.7312		0.7542	
χ^2 (after pooling) =	3.56		0.08		0.91	
$\hat{p}$ =	0.8327		0.8190		0.8180	
p-value=	0.0591		0.9607		0.6344	
d.f=	1		2		2	

Table 4. Observed and Expected number of households having adult Male Migrants aged 15 years and above.		
Number of migrants	**Observed no. of households**	**Expected no. of households**
1	97	98.90
2	42	40.33
3	16	16.44
4	7 } 12	11.33
5+	5	
Total	**167**	**167**
Mean=1.68 Variance=1.142 $\hat{p}$ =0.7695	χ^2 = 0.13 (after pooling) p-value=0.9370 $\chi^2_{(2)}$ = 5.99 at 5% level of signi icance	

Table 5. The number of mothers of the Rural Area having at least one live birth and one neonatal death.		
Number of Neonatal deaths	**Observed no. of mothers**	**Expected no. of mothers**
1	409	408.7
2	88	88.65
3	19	19.22
4	5 } 6	5.42
5	1	
Total	**522**	**522**
Mean=1.27 Variance=0.3538 $\hat{p}$ =0.8849	χ^2 = 0.067 (after pooling) p-value=0.9670 $\chi^2_{(2)}$ = 5.99 at 5% level of signi icance	

Table 6. The number of mothers of the Estate Area having at least one live birth and one neonatal death.		
Number of Neonatal deaths	**Observed no. of mothers**	**Expected no. of mothers**
1	71	72.90
2	32	27.86
3	7	10.64
4	5 } 8	6.59
5	3	
Total	**118**	**118**
Mean=1.618 Variance=1.001 $\hat{p}$ =0.7860	χ^2 = 2.19 (after pooling) p-value=0.3345 $\chi^2_{(2)}$ = 5.99 at 5% level of signi icance	

Table 7. The number of mothers of the Urban Area with at least two live births by the number of infant and child deaths.

No. of Infant and child deaths	Observed no. of mothers	Expected no. of mothers
1	176	172.06
2	44	50.72
3	16	14.95
4	6 } 8	6.27
5	2	
Total	**244**	**244**

<table>
<tr><td>Mean=1.41
Variance=0.5930
$\hat{p}$ =0.8397</td><td>χ^2 = 1.53 (after pooling)
p-value=0.4653
$\chi^2_{(2)}$ = 5.991 at 5% level of signi icance</td></tr>
</table>

Table 8. The number of mothers of the Rural Area with at least two live births by the number of infant and child deaths.

No. of Neonatal deaths	Observed no. of mothers	Expected no. of mothers
1	745	739.99
2	212	212.45
3	50	60.99
4	21	17.51
5	7 } 10	7.06
6	3	
Total	**1038**	**1038**

<table>
<tr><td>Mean=1.40
Variance=0.5651
$\hat{p}$ =0.8443</td><td>χ^2 = 3.92 (after pooling)
p-value=0.2702
$\chi^2_{(3)}$ = 7.815 at 5% level of signi icance</td></tr>
</table>

Table 9. The number of literate mothers with at least one live birth by the number of infant deaths.

No. of Neonatal deaths	Observed no. of mothers	Expected no. of mothers
1	683	675
2	145	153
3	29	34.7
4	11 } 16	10.3
5	5	
Total	**873**	**873**

<table>
<tr><td>Mean=1.29
Variance=0.3794
$\hat{p}$ =0.8793</td><td>χ^2 = 4.72 (after pooling)
p-value=0.0944
$\chi^2_{(3)}$ = 5.99 at 5% level of signi icance</td></tr>
</table>

Table 10. The number of mothers of the completed fertility having experienced at least one child death.		
No. of child deaths	**Observed no. of mothers**	**Expected no. of mothers**
1	89	83.98
2	25	31.73
3	11	11.99
4	6	
5	3 } 10	7.30
6	1	
Total	**135**	**135**
Mean=1.60 Variance=0.9767 $\hat{p}$ =0.7887	χ^2 = 2.80 (after pooling) p-value=0.2465 $\chi^2_{(2)}$ = 5.99 at 5% level of signi icance	

Table 11. The number of mothers having at least one neonatal deaths.		
No. of Neonatal deaths	**Observed no. of mothers**	**Expected no. of mothers**
1	567	562.13
2	135	138.56
3	28	34.15
4	11 } 16	11.15
5	5	
Total	**746**	**746**
Mean=1.32 Variance=0.4342 $\hat{p}$ =0.8680	χ^2 = 3.33 (after pooling) p-value=0.1891 $\chi^2_{(2)}$ = 5.99 at 5% level of signi icance	

Fig.1 Graphical presentation showing expected and observed number of household aged 15 years and above (survey 2001 data)

Fig.2 Graphical presentation showing expected and observed number of household aged 15 years and above (survey 1978 data)

Fig.3 Graphical presentation showing expected and observed number of household aged 15 years and above (survey 1978 data) in three types of households.

Fig.4 Graphical presentation showing Observed and Expected number of households having adult Male Migrants aged 15 years and above

Fig.5 Graphical presentation showing Rural Area having at least one live birth and one neonatal death.

Fig.6 Graphical presentation showing Estate Area having at least one live birth and one neonatal death

Fig.7 Graphical presentation showing Urban Area with at least two live births by the number of infant and child deaths

Fig.8 Graphical presentation showing Rural Area with at least two live births by the number of infant and child deaths.

Fig.9 Graphical presentation showing literate mothers with at least one live birth by the number of infant deaths.

Fig.10 Graphical presentation showing mothers of the completed fertility having experienced at least one child death

Fig.11 Graphical presentation showing mothers having at least one neonatal deaths

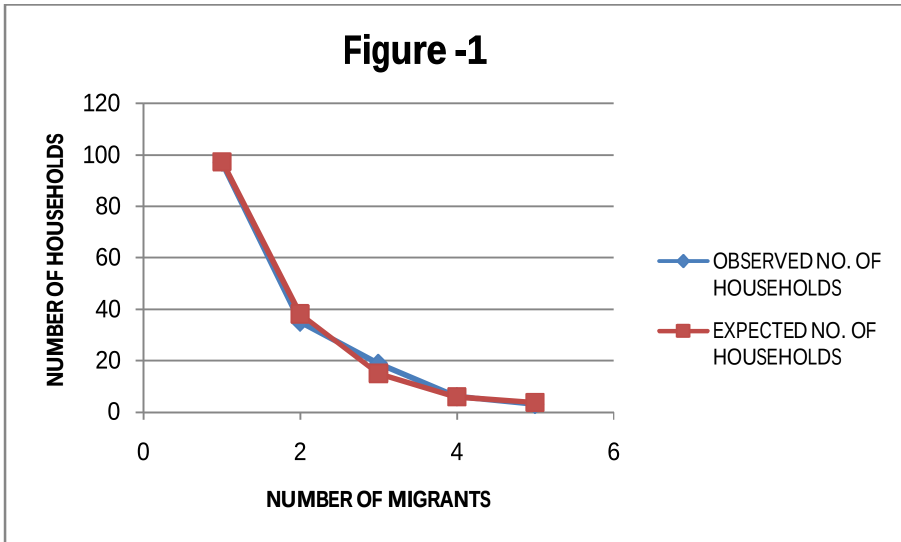
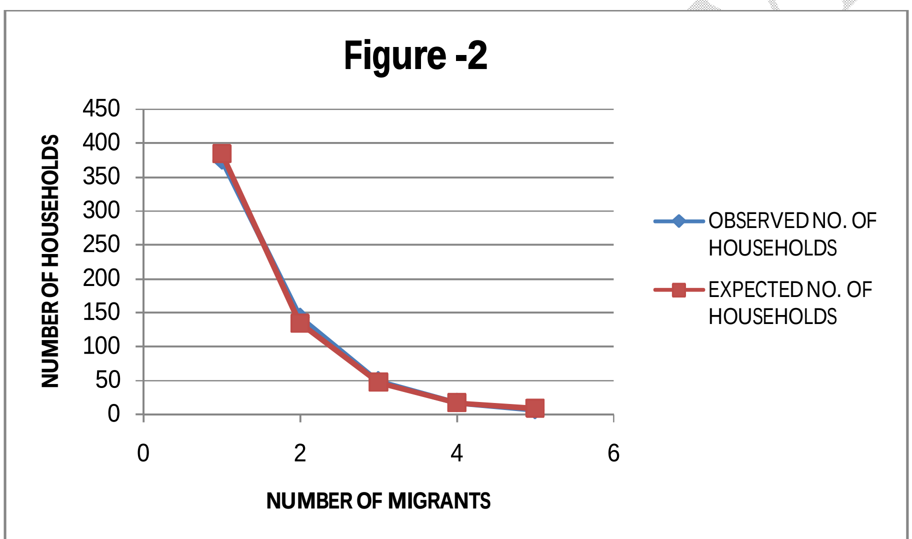


Figure -1
NUMBER OF HOUSEHOLDS
120
100
80
60
40
20
0
0
2
4
6
NUMBER OF MIGRANTS
OBSERVED NO. OF HOUSEHOLDS
EXPECTED NO. OF HOUSEHOLDS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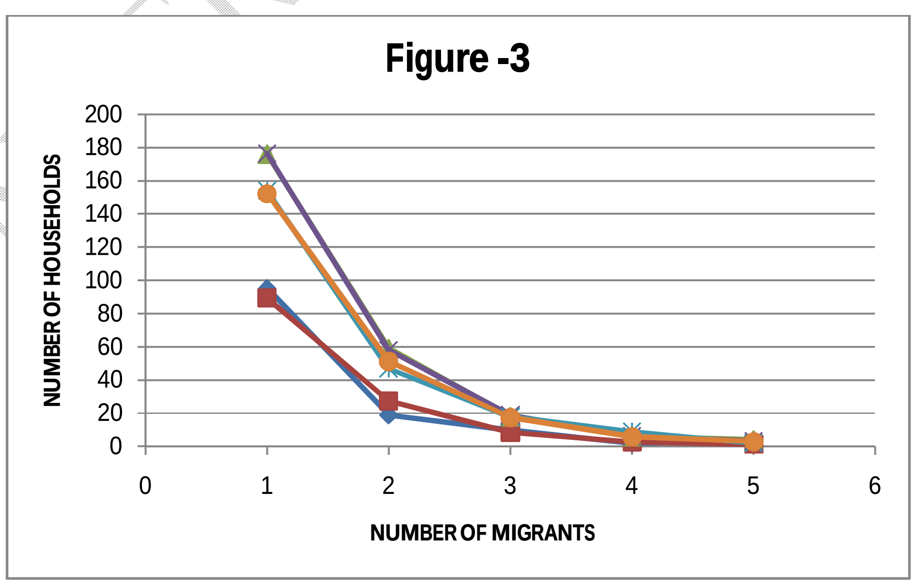
NUMBER OF HOUSEHOLDS
450
400
350
300
250
200
150
100
50
0
0
2
4
6
NUMBER OF MIGRANTS
OBSERVED NO. OF HOUSEHOLDS
EXPECTED NO. OF HOUSEHOLDS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NUMBER OF HOUSEHOLDS
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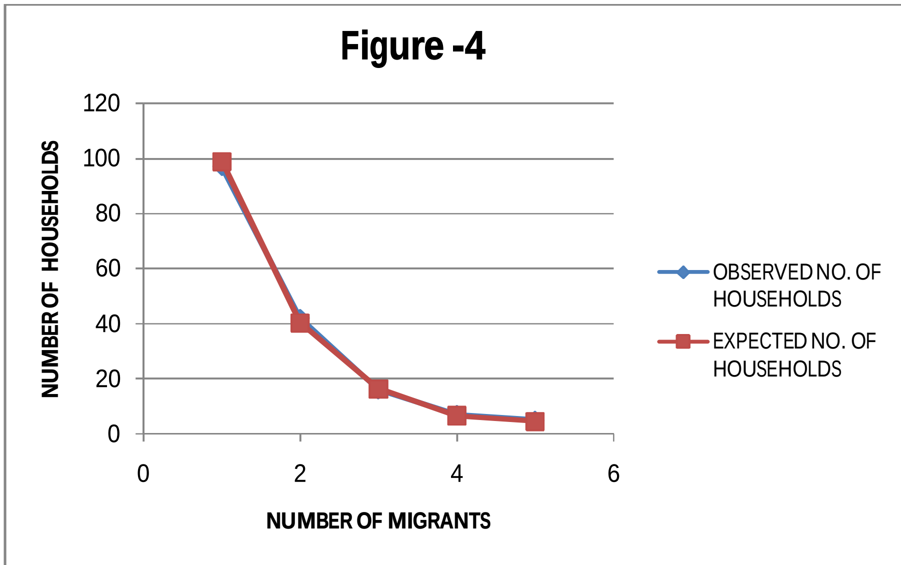
140
120
100
80
60
40
20
0
0
1
2
3
4
5
6
NUMBER OF MIGRANTS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80
60
40
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2
4
6
NUMBER OF HOUSEHOLDS
NUMBER OF MIGRANTS
OBSERVED NO. OF HOUSEHOLDS
EXPECTED NO. OF HOUSEHOLDS

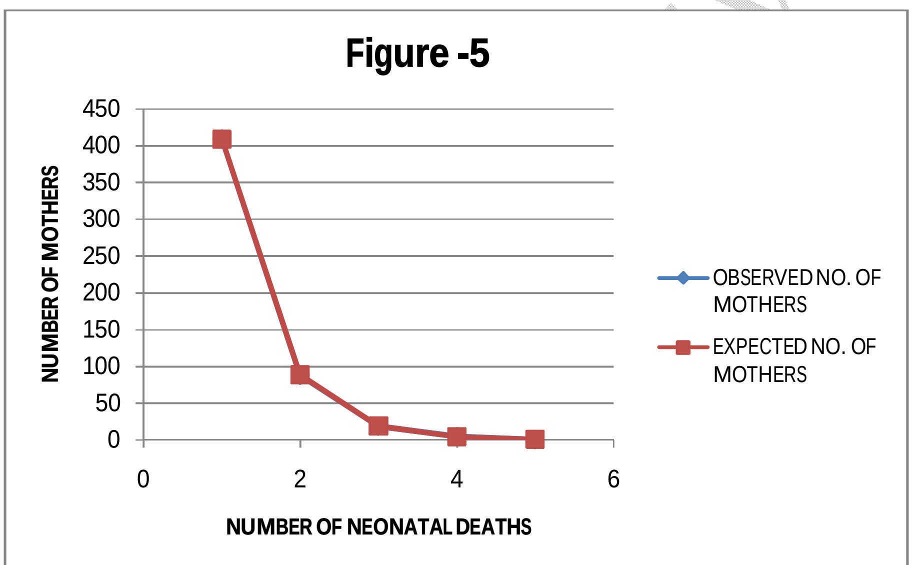
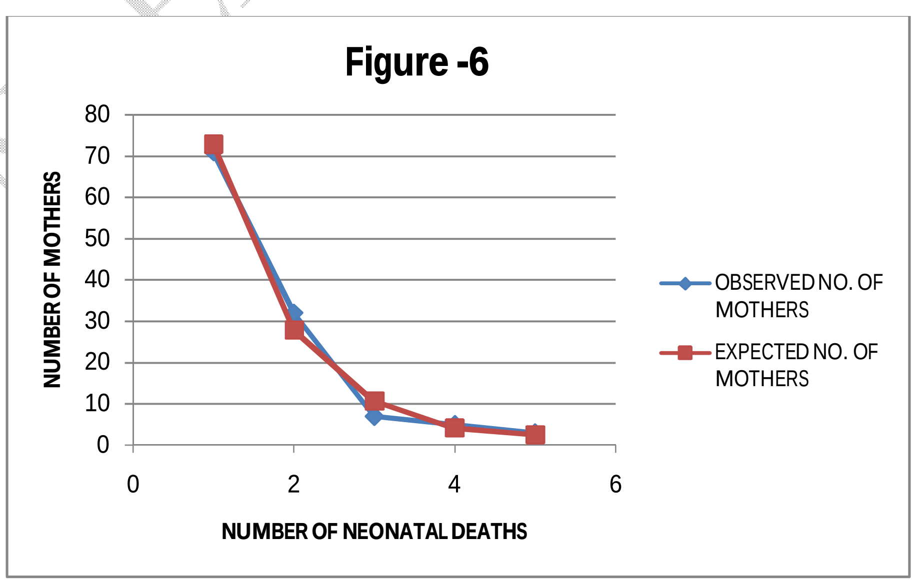


Figure -5
450
400
350
300
250
200
150
100
50
0
0
2
4
6
NUMBER OF MOTHERS
NUMBER OF NEONATAL DEATHS
OBSERVED NO. OF MOTHERS
EXPECTED NO. OF MOTHERS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70
60
50
40
30
20
10
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2
4
6
NUMBER OF MOTHERS
NUMBER OF NEONATAL DEATHS
OBSERVED NO. OF MOTHERS
EXPECTED NO. OF MOTHERS

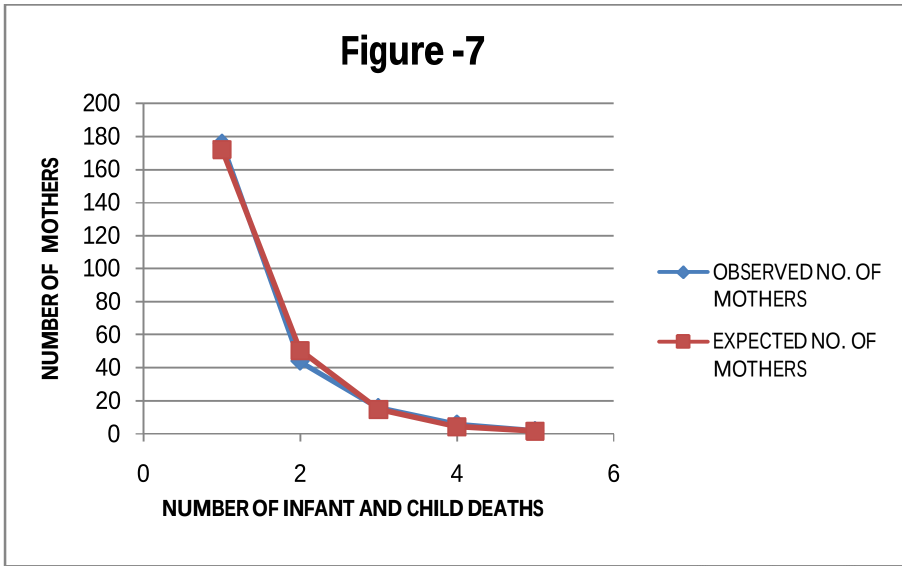


Figure -7
NUMBER OF MOTHERS
200
180
160
140
120
100
80
60
40
20
0
0
2
4
6
NUMBER OF INFANT AND CHILD DEATHS
OBSERVED NO. OF MOTHERS
EXPECTED NO. OF MOTHERS

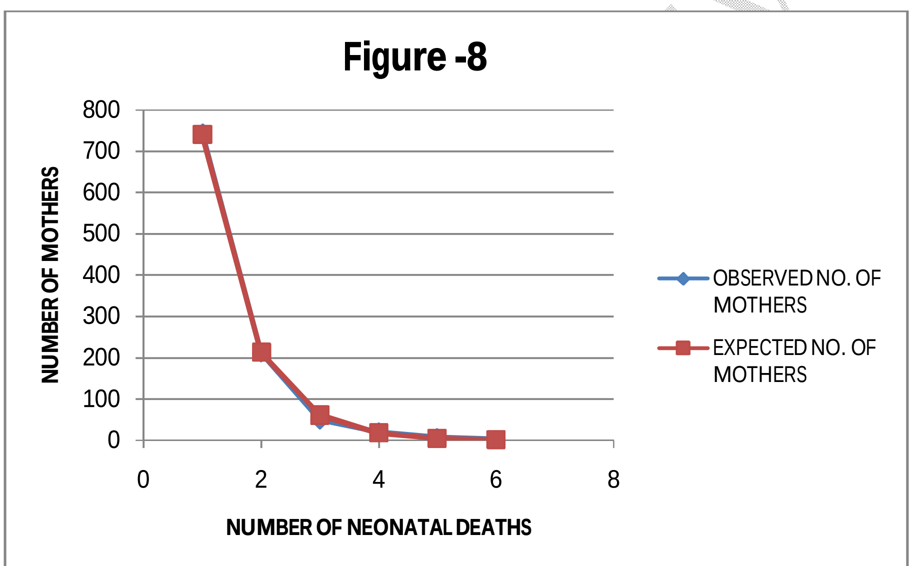
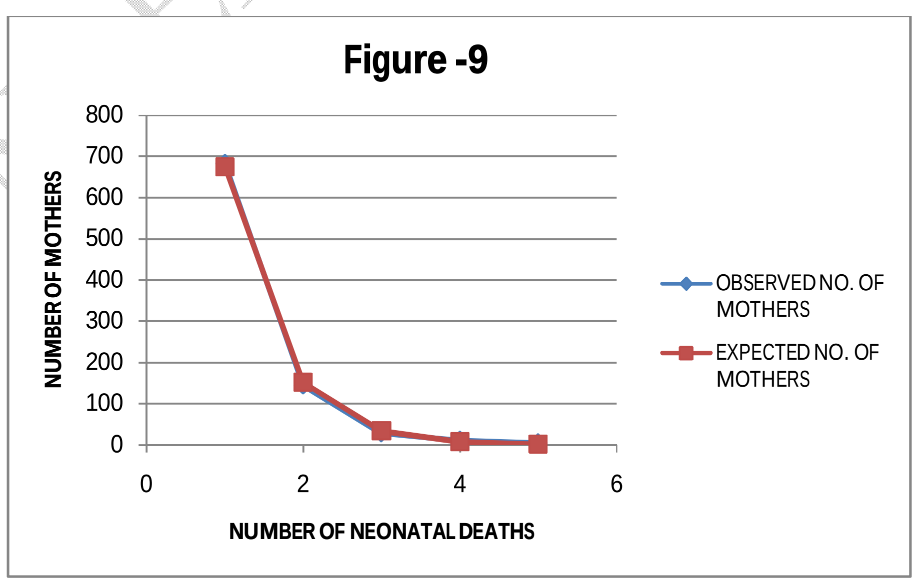


Figure -8
NUMBER OF MOTHERS
800
700
600
500
400
300
200
100
0
0
2
4
6
8
NUMBER OF NEONATAL DEATHS
OBSERVED NO. OF MOTHERS
EXPECTED NO. OF MOTHERS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NUMBER OF MOTHERS
800
700
600
500
400
300
200
100
0
0
2
4
6
NUMBER OF NEONATAL DEATHS
OBSERVED NO. OF MOTHERS
EXPECTED NO. OF MOTHERS

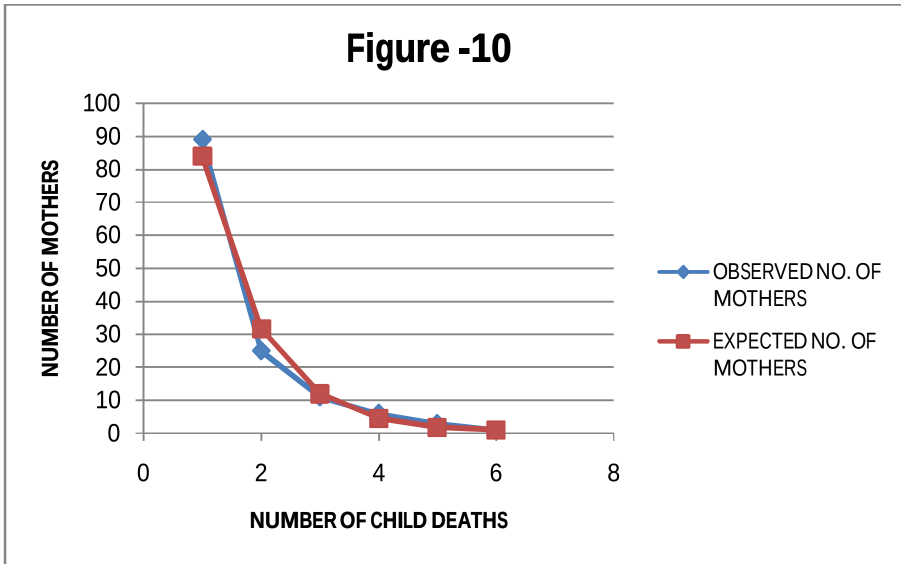
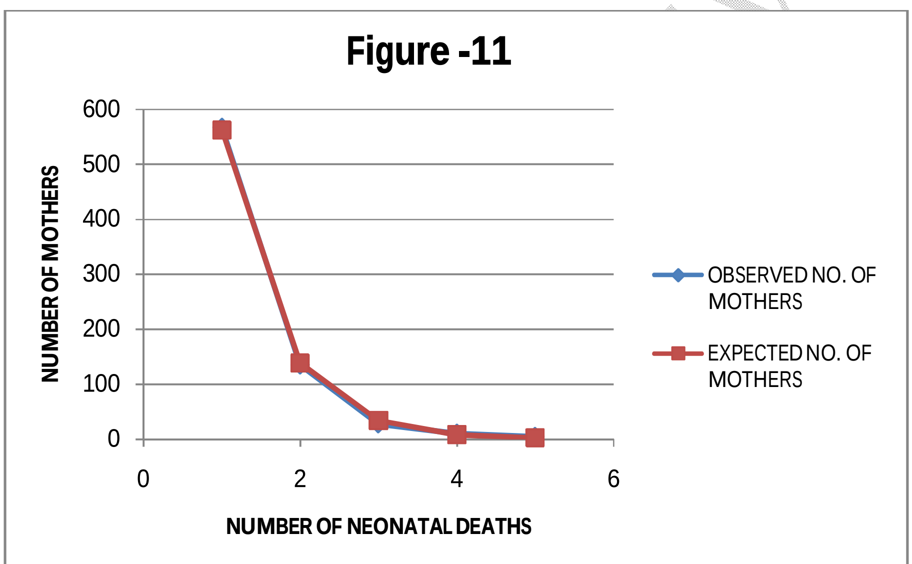


Figure -10
NUMBER OF MOTHERS
100
90
80
70
60
50
40
30
20
10
0
0
2
4
6
8
NUMBER OF CHILD DEATHS
OBSERVED NO. OF MOTHERS
EXPECTED NO. OF MOTHERS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NUMBER OF MOTHERS
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500
400
300
200
100
0
0
2
4
6
NUMBER OF NEONATAL DEATHS
OBSERVED NO. OF MOTHERS
EXPECTED NO. OF MOTHERS

5. Conclusion

According to the value of χ^2 and p-value From (table 1,2,3,4,5,6,7,8,9,10,11) and graphical representation between O_i and E_i, the nature and behavior of proposed ZTHD Model found suitable for the migration pattern as well as infant mortality pattern of different Regions.

The overall studies shows that the proposed ZTHD Model could also be helpful in policy making, Rural development, Fresh Environment, Medical Facilities for the betterment of the society. In the light of work done by Rao and Pandey (2017,2019), it is also possible for the Bayesian Analysis of the proposed model.

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