Minireview Article

The distances between, and the formation times of, the galaxies

Abstract.

Since the James WebsSpace Telescope, JWST, came into operation during the first half of the year 2022, cosmological times and distances have been reported, that reach more than 13 billion years and light years. These cosmological data are of a much higher order of magnitude, than those handled in traditional astronomy. When analyzing the information published by Naidu, R. P. etal. and other authors, about the Glass galaxy z13, GZ, recently discovered by the JWSP, where a redshift, z = 13.1, of the captured signal was measured; it is concluded that GZ is being seen in the form and position in which it was 13,500 million years ago. During this time the light traveled a distance, Dv, equivalent to 13.5 billion light years, for now, been detected in the Milky Way, MW. Next, it is explained what this data means and how it is possible to calculate: 1). The distance,Da, the two galaxies (MW and GZ) are now, 2). The distance, Do, at which the said galaxies were located 13,500 million years ago, 3). The distance, Dm, that each of the galaxies moved, towards opposite directions, 4). The scale factor, a(t), and the relation of the commoving distance, to the proper distance y 5). Comparison of the above-mentioned cosmological distances, with other local astronomical distances such as the distance between the MW and the Andromeda galaxy and finally the distance between the Earth and a planet, but that of the closest star to the Sun, the planet Proxima Centauri b, described by Davis, N. K. S. These comparisons are intended to get a better idea of the orders of magnitude that cosmological distances and astronomical distances mean.

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Keywords: Glass galaxy z13, distances between galaxies.

1). The current distance, Da, that the MW is from the GZ.

Hubble's law says that the speed, V, of recession or separation between one galaxy from another, is equal to the Hubble constant, Ho, multiplied by the distance, D, between the two galaxies. If the initial separation distance is Do and the current separation after a time, t, is the distance, Da, then it can be formulated that the average speed, at which each of the galaxies moved in opposite directions, is equal to one-half, of the sum of the initial speed, Ho Do, and final speed, Ho Da. And also that, the distance that each of the galaxies moved in opposite directions, Dm, is equal to the average speed multiplied by the time, t:

$$\text{Dm} = \left(\frac{(HoDo + HoDa)}{2}\right)(t) \quad \ldots(1)$$

Twice the moving distance, Dm, see Figure 1, added to the initial distance, Do, equals the current distance, Da, that the two galaxies are located:

$$\text{Da} = (HoDo + HoDa)(t) + \text{Do} \quad \ldots(2)$$

Expanding equation (2) to clear Da from there, we get:

$$Da = Ho\ Do\ t + Ho\ Da\ t + Do \quad \ldots (3)$$

$$Da\ (1 - Ho\ t) = Do\ (1 + Ho\ t) \quad \ldots (4)$$

$$Da = Do\ \frac{(1 + Hot)}{(1 - Hot)} \quad \ldots (5)$$

In equation (5) Da is expressed as a function of Do. Next, an expression for Do is derived, based on the data obtained by visualizing a galaxy and measuring its redshift, that is Dv.~~In~~ Figure 1 are showed the distances relationships to be used.From this Figure,it can be established that Da is equal to Do, plus twice, the distance traveled, Dv, minus Do, that is:

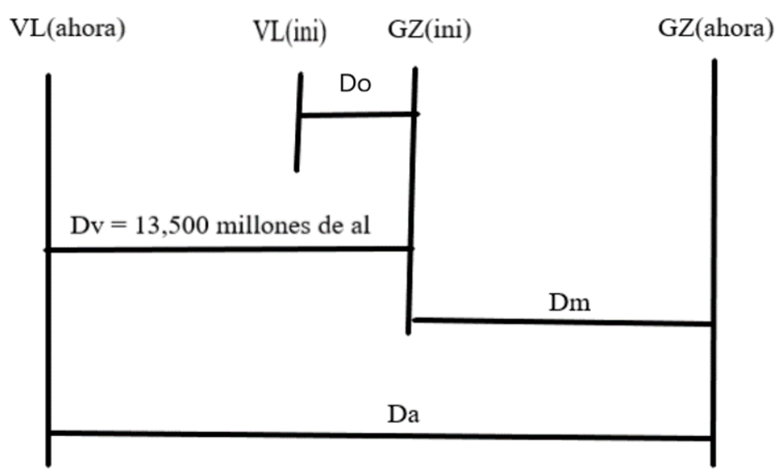

Figure 1Distance relationships

$$Da = Do + 2\ (Dv - Do) \ldots (6)$$

$$Da = Do + 2\ Dv - 2\ Do = 2\ Dv - Do \ldots (7)$$

From equation (7), Do is cleared as:

$$Do = 2\ Dv - Da \quad \ldots (8)$$

Substituting equation (8) in equation (5) we have:

$$Da = (2\ Dv - Da)\frac{(1 + Ho\ t)}{(1 - Ho\ t)} \quad \ldots (9)$$

from the expansion of equation (9) it is cleared Da, as a function of Dv:

$$Da - Da\ Ho\ t = 2\ Dv - Da + 2\ Dv\ Ho\ t - Da\ Ho\ t \quad \ldots (10)$$

$$2\ Da = 2Dv + 2Dv\ Ho\ t \quad \ldots (11)$$

$$Da = Dv + Dv\ Ho\ t \quad \ldots (12)$$

Since the distance traveled by light is equal to the speed of light, c, times the time traveled, t, that is:

$$Dv = c\ t \quad \ldots(13)$$

~~From~~E~~e~~quation (13) it is cleared the time:

$$t = Dv\ /\ c \quad \ldots(14)$$

Now the value of time from equation (14) is substituted into equation (12) ~~in order~~ to obtain:

$$Da = Dv + (Ho\ /\ c)\ Dv^2 \quad \ldots(15)$$

Next, Dv, Ho, and t are obtained in compatible units.

The speed of light (300,000 Km/sec) is now expressed in Km/year according to:

$$c = (300{,}000\ Km/sec)\ (3.154 \times 10^7\ sec/year) = 9.4693 \times 10^{12}\ Km/year \ldots(16)$$

As an ~~example~~example,~~,~~ it can be calculated that the 13,500 million ly, that the light traveled, since it left the initial position of VZ until reaching MW, are equal to the particular Dv of:

$$Dv = (13{,}500 \times 10^6 ly)\ (9.4693 \times 10^{12}\ km/ly) = 1.2784 \times 10^{23}\ km \quad (17)$$

About the Hubble constant, the following is commented on it: it is known that, according to the technique used in its determination, different values are obtained, which range, according to Skibba, R. between 60 and 80 (km/sec)/Mpc. For this article, the value of 71.0 (km/sec)/Mpc will be used. Now, as a megaparsec, Mpc, is equal to 3.087×10^{19} Km, then Ho is equal to:

$$Ho = (71.0\ km/sec)/(3.087x10^{19}\ Km) = 2.30x10^{-18}\ sec^{-1} \quad ...\ (18)$$

The advantage obtained with this value of Ho is that if it is used to calculate the age of the Universe (1/Ho), also called the Hubble time, according to Hawley, J. F. et al., a value of Hubble time equal to 13.8 billion years is obtained, which it is the value for the age of the Universe, commonly accepted by most cosmologists:

$$1/Ho = 1/(2.30x10^{-18}\ sec\text{-}1) = 4.3535x10^{17}\ sec$$
$$= (4.3535x10^{17}\ sec)/(3.154x10^{7}\ sec/year) = 1.38x10^{10}\ years$$
$$= 13,800\ millions\ of\ years\ ...(19)$$

Now, following the example of GZ, it is calculated that the time, t, expressed in seconds, that elapsed in the 13.5 billion years in which light traveled from GZ to MW, is equal to:

$$t = (13,500x10^{6}\ years)(3.154x10^{7}\ sec/year) = 4.2579x10^{17}\ sec \quad ...\ (20)$$

With the value of the constant Ho expressed in equation (18), and the value of the speed of light, c, expressed in equation (16), it is solved in a generalized form, equation (15) for Da, in the interval of Dv = [500 to 13,500] million light years. The result is shown in Figure 2.

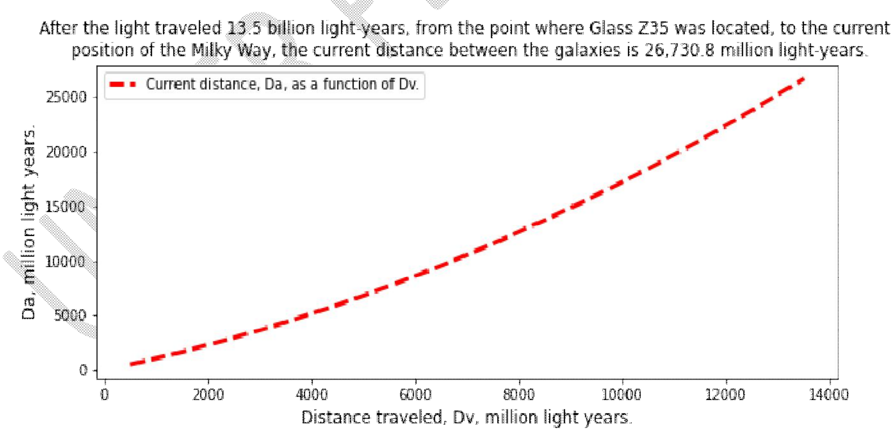

Figure 2. Distance between galaxies in light years

2). Distance, Do, to which the GZ galaxy and the point where the MWgalaxywas going to form, $13,500x10^{6}$ years ago.

To calculate the initial distance, Do, between the GZ galaxy and the point where the MWgalaxywas going to form, in which the light rays came out of the first one, andafter a time, t, they were detected by the second galaxy, use is made of the concept designated as retrospective time, which consists of accepting that the further one looks into space, the further one goes back in time.

Next, Do is expressed as a function only of the time, t, that the light has traveled from the source galaxy to the receiving galaxy. Starting from equation (12):

$$Da = Dv + Dv\ Ho\ t \quad ...(12)$$

Now the value of Da expressed by equation (5) is taken:

$$Da = Do\ \frac{(1 + Hot)}{(1 - Hot)} \quad ...(5)$$

And equation (12) is equated with (5) to obtain:

$$Do\ \frac{(1 + Hot)}{(1 - Hot)} = Dv + Dv\ Ho\ t \quad ...\ (21)$$

Next, the value of the distance traveled by light, Dv, which has already been expressed in equation (13), is taken:

$$Dv = c\ t \quad ...(13)$$

Substituting equation (13) in equation (21) gives:

$$Do\ \frac{(1 + Hot)}{(1 - Hot)} = c\ t + c\ Ho\ t^2 \quad ...\ (22)$$

When developing and simplifying equation (22) to clear for Do, we obtain:

$$Do + Do\ Ho\ t = c\ t + c\ Ho\ t^2 - c\ Ho\ t^2 - c\ Ho^2\ t^3 \quad ...(23)$$

$$Do\ (1 + Ho\ t) = c\ t + - c\ Ho^2\ t^3 \quad ...(24)$$

$$Do = (c\ t - c\ Ho^2\ t^3)/(1 + Ho\ t) \quad ...(25)$$

Taking the value of Ho from equation (18) and the value of the speed of light, c, from equation (16), the value of Do is obtained as a function of the light traveled time, t, in the interval from 500 to 13,500 million years. The solution of equation (25) is shown graphically in Figure 3.

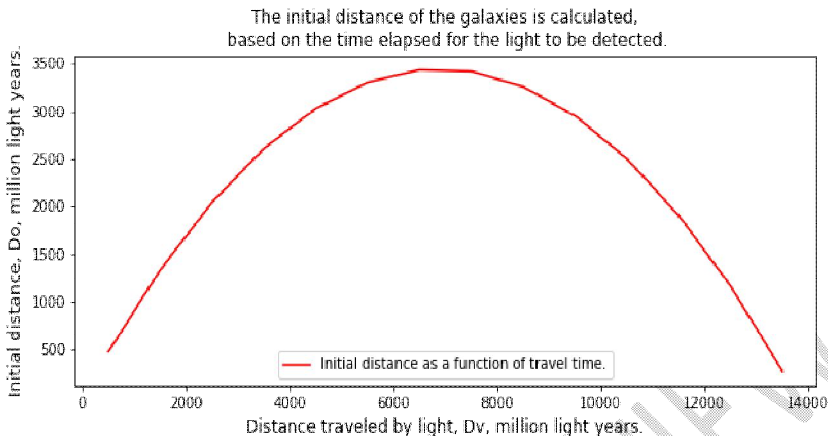

Figure 3.Initial distance of galaxies

When Do is calculated, with equation (25), for a travel time of 13,500 years, it is obtained:

$$Do = 220.8 \times 10^{6} \text{ ly} = 220.8 \text{ million light years} \ldots \quad (25)$$

In Figure 3, it also can be seen, that both for t = 0, and for a time equal to the Hubble time of existence of the universe, (The one in equation (19), t = 13,800 million lightyears), the initial distance Do = 0.0. This can be interpreted as~~:~~ a). If the time is zero, it is because the light traveled directly from one galaxy to another, separated by a zero distance, and b). If the time is equal to the Hubble time of existence of the universe, it is because the light traveled in the opposite direction to which the two galaxies were with zero separation.

Regarding the results of the initial distance shown in Figure 3, at which the galaxies were separated, the following comments can be made: a). The maximum separation, or initial distance, at which two galaxies can be separated is 3.4 billion light-years, which comes from exactly half the Hubble time (6.9 billion years) of the existence of the universe, b). When the travel time is between 0.0 and 6.9 billion years, the initial separation distance increases. This can be interpreted as, the light travels from the light-emitting galaxy, directly to the receiving galaxy of said light, and c). When the travel time is between 6.9 and 13.8 billion years, the initial separation distance decreases. This ~~Which~~ can be interpreted as that the light travels from the light-emitting galaxy to the receiving galaxy in the opposite direction that they are in, so that, although the light travel time is increasing, the initial distance between the two galaxies is decreasing.

3). Distance, Dm, that the MW and GZ traveled, in opposite directions, from the point the GZgalaxywas located, to the pointwhere the MW was going to be.

Each galaxy moved the distance Dm, see Figure 1, which is equal to one-half the difference between the current separation distance, Da, and the distance, Do, between VZ and the point where the MW formed, that is:

$$Dm = (1/2)(Da - Do) \quad \ldots (26)$$

When the distances of Da and Do are obtained as a function of the traveled distance,as in sections 1 and 2 above, then equation (26) is solved directly. Figure 4 shows the values of the distances that the galaxies moved as a function of the traveled time, t, in millions of years, and whose numerical value is the same as that of the travel distance, Dv, of the light.

When equation (15) is solved for Dv = 13,500 million light years, the value of Da = 26,779.2 million light years is obtained. Substituting this value and the value of Do that was obtained in equation (25), we obtain:

$$Dm = (½)(26,779.2x10^6 \text{ ly} - 220.8x10^6 \text{ ly}) \quad ... (27)$$

$$Dm = 13,293.2x10^6 \text{ ly} \quad ... (28)$$

==============

This particular solution can be visualized in Figure 4 below, where equation (26) is solved in a generalized form, for a Da and a Do calculated as a function of the travel time of the light, between the emitting galaxy and the light-receiving galaxy.

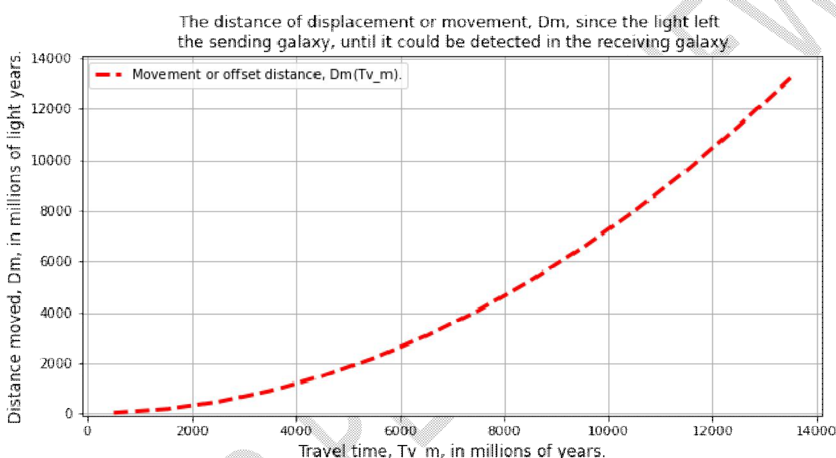

Figure 4. Distance of displacement

4). The scale factor a(t), the proper distance, D(t), and the co-movement distance, Dcm.

According to Paolera, C. D., in Cosmology ~~there are~~ two distances ~~that~~ are of great interest: a). The proper distance, D(t), which is the distance between the emitting and receiving galaxies, taking into account the universal expansion and the time that the light used to travel from the emitting galaxy to the receiving galaxy, and b). The distance by co-movement, Dcm, ~~which~~ is the distance that the galaxies have at the moment they are observed.

Following the exposition of the link "https://explainingscience.org", it is found that the previous two distances are correlated with the scale factor, a(t), according to the following equation:

$$Dcm = D(t)/a(t) \quad ...(29)$$

If it is accepted that nowadays, a(t) =1, then the current distance, Da, calculated in equation (12) is equivalent to the proper distance, D(t), just defined:

$$D(t) = Da = Dv + \left(\frac{Ho}{c}\right) Dv^2 \quad \ldots (30)$$

Also, in this case, the co-movement distance, Dcm, from equation (29) would be equal to D(t).

For the case in which the value of the scale factor, a(t) ≠ 1.0, then combining equations (29) and (30) we obtain:

$$Dcm = \frac{1}{a(t)} \left(Dv + \left(\frac{Ho}{c}\right) Dv^2\right) \quad \ldots (31)$$

One of the possible causes that a(t) ≠ 1.0 is that one or the two galaxies (emitter and/or receiver) that are being analyzed is gravitationally attracted by a cluster of galaxies, with which the Dcm can be smaller or larger than the Da expressed in equation (12).

Figure 5 shows the solution of equation (31) for the values of a(t) = [0.6, 0.8, 1.0, 1.2].

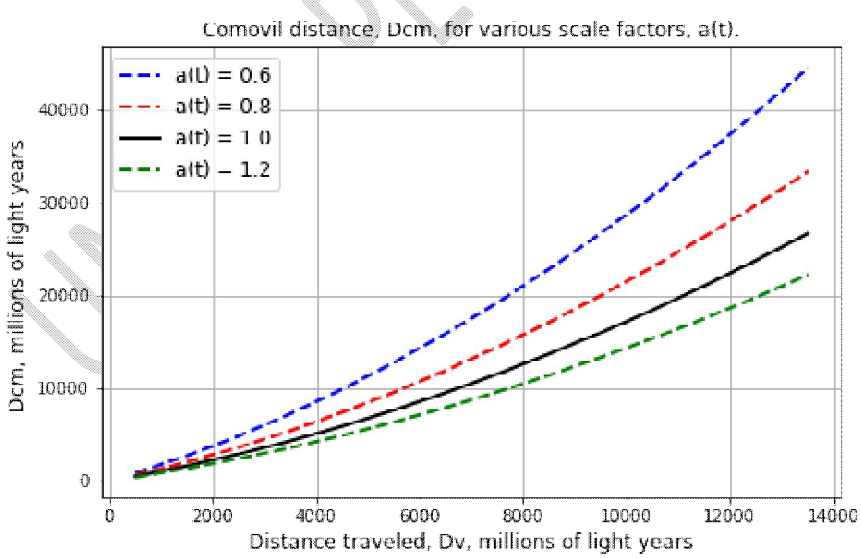

Figure 5.Comovil distance

When using the Kempner, J. Cosmological Calculator, it has been reported that the actual distance between MW and GZ is 33,205 million light years. According to what has been exposed, this distance corresponds to a scale factor, a(t) = 0.805, for a Dv = 13,500 million light years.

5). Reference astronomical distances, for described cosmological distances.

Next, two examples will be described, as reference points ~~in order~~ to understand the cosmological distances of Da and Do that have been calculated, when they are related to two astronomical distances.

The first example is that, of the current distance between the MW and the Andromeda galaxy, which is known to be 2.5 million ly, at this distance, the galaxies are no longer moving apart, but instead they are gravitationally attracted. Since the MW and the Andromeda galaxy are relatively close, it has been calculated that these two galaxies are approaching each other at a speed of around 300 kilometers per second~~,~~ and that they will start colliding in about 3,800 million years, to end up fused in 5,860 million years.

Regarding the initial distance between VZ and MW, it should be noted that the value of Do obtained is 88 times greater than the current distance between VL and Andromeda. ~~And that,~~ ~~F~~furthermore, the MW never had the possibility of being gravitationally attracted by VZ. Since the MW began to form around 12.6 billion years ago, that is, 900 million years after the departure of the light that now reaches the MW. This~~Which~~ means that when the MW began to form there was a distance of $900x10^6$ ly + $220x10^6$ ly = $1,120.6x10^6$ ly, between GZ and the incipient MW.

On the other hand, if the distance traveled, Dv, between VZ and MW, had been only 500 million light years, the distances Da and Do ~~obtained~~obtain by repeating the calculations of equations (12) to (25), have the following values:

Da = 518.2 million ly.

Do = 481.8 million ly.

and that can be roughly visualized in Figures 2 and 3.

The second comparative example is that of the distance between the Sun and the closest star called Proxima Centauri, which is only 4.2 light years. This distance is on the order of hundreds of millions of times smaller than the cosmological distances that have already been described. However, an interstellar trip between the planet earth and the planet Proxima Centauri b, from the previous star, is still impossible for humanity. Since the 4.2 ly that separates them is~~are~~ equivalent to 3.977x10^{13} km (39.77 million~~ million~~ km). The fastest spacecraft ever built by humans, the Parker Space Probe, which is expected to move at 200 km/s, would take 6,300 years to reach the planet Proxima Centauri b.

$$\frac{3.977*10^{13}\,km}{200\ km/sec} = 1.988\text{x}10^{11}\text{ sec} = \frac{1.988*10^{11}\ sec}{3.154*10^{7}\ sec/year} = 6.3\text{x}10^{3}\text{ years} \quad ...(32)$$

Hence, there is a lot to develop~~ by~~ the human~~s~~ about technologies that allow at least interstellar flights (Since intergalactic flights at this time are seen as impossible). The spacecraft~~s~~ that can probably be developed must be able to travel at speeds close to the speed of light, or 300,000 km/s. If ~~a~~technology were developed for a spacecraft to fly at one-tenth the speed of light, it would take 42 years for this spacecraft to reach Proxima Centauri b. From the above, it is seen that the relatively short distances between stars are~~ really~~ very large. And that the distances between galaxies are extremely large.

Conclusions.

When the JWST telescope detects a galaxy and it can be determined how many years ago the light now seen left that galaxy, then the proper distance, D(t), and the comoving distance, Dcm, can be determined, to which the Milky Way galaxy, is currently located. Both galaxies are supposed to have moved in opposite directions according to Hubble's law. It is also possible to calculate the initial distance, Do~~,~~ at which the galaxies were separated when the light from one of them left, and that is being captured in the MW.

In the case of the detection by the JWST of the galaxy, which at the time was taken as one of the oldest galaxies in the universe, Glass Z13, GZ, and which is being seen with the shape and position it was 13.5 billion years ago; it was calculated: a). The proper distance D(t) or current Da for a scale factor, a(t) = 1.0, b). The comovil or comovement distance, Dcm, between the MW and the GZ~~,~~ which corresponds to a(t) = 0.805, c). Its initial distance, Do, when the light left GZ, and d). The distance that each of these galaxies traveled, Dm, in opposite directions. The results are:

D(t) = Da = 26,730.8x10^{6}ly

Dcm = 33.206x10^{6} ly

Do = $220.8x10^6$ly

Dm = $13{,}293.2x10^6$ly

In addition, the generalized calculation of Dcm was made, as a function of Dv for four values of a(t). The results are shown in Figures 2, 3, 4, and 5.

The appendix shows the 4 programs written in Python, on the Anaconda Jupiter Notebook platform, to make the generalized calculations and the graphs of the figures mentioned. The 4 programs shown can be copied and then pasted into cells of Jupiter Notebook. It has already been tested that they run without errors. It is ~~also~~ possible to see in there, not only the graphs but also, the numerical values of the solutions.

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Appendix.

```
# 1. CALCULATION OF THE CURRENT DISTANCE, Da, AT WHICH THE
GALAXIES VL
# AND VZ ARE lOCATED, AS A FUNCTION OF THE DISTANCE
TRAVELLED, Dv.

import numpy as np
import scipy as sp
%matplotlib inline
import matplotlib.pyplot as plt

Dv_al = np.linspace(500*pow(10,6),13500*pow(10,6),14)
        # Dv_al = Distance traveled in light years.

Dv_km = (Dv_al)*(9.4693*pow(10,12))
        # Dv_km = Distance traveled in km. (al)*(km/al) = km
Ho = (71.0)/(3.087*pow(10,19))     # Hubble constant, (km/seg)/(Km)
= 1/sec
print("Ho = ",round(Ho,22)," 1/sec")
print()

c = 3e5
Da = Dv_km + (Ho/c)*(Dv_km**2)

Da_al = Da/(9.4693*pow(10,12))     # (km)/(km/al) = ly = light years
Da_m = Da_al/1000000               # Da_m = Current distance of
galaxies
# in millions of light years.
Dv_m = Dv_al/1000000               # Dv_m = Distance traveled by light
                                   # in millions of light years.

print("Distance traveled, Dv, in millions of ly.")
```

```
print(Dv_m)
print()
print("Current distance, Da, between galaxies, in millions of ly.")
print(Da_m)

plt.figure(figsize = (10,4))
plt.plot(Dv_m,Da_m,'r--',linewidth=3)
plt.legend(["Current distance, Da, as a function of Dv."])
plt.xlabel('Distance traveled, Dv, million light years.', fontsize=12)
plt.ylabel('Da, million light years.', fontsize=12)
plt.title("After the light traveled 13.5 billion light-years, from the point
where Glass Z35 was located, to the current \n position of the Milky
Way, the current distance between the galaxies is 26,730.8 million light-
years.")
plt.savefig('Da(Dv).png')


# 2. CALCULATION OF THE INITIAL DISTANCE, Do, AT WHICH THE GALAXIES
# WERE FOUND DEPENDING ON THE TRAVEL TIME OF LIGHT.

import numpy as np
from scipy.optimize import fsolve

t_año = 3.154e7  # sec/year
Tv_a = np.linspace(0.5e9,13.5e9,14)
# Tv_a = Time traveled in billions of light years.
print("Time traveled in billions of light years.")
print(Tv_a)
Tv = Tv_a*t_año  # Tv, time traveled in sec. since: (years)*(sec/year) = sec.

print()
c = 3e5          # km/sec
Ho = 2.30e-18      # 1/sec

Do = (c*Tv - c*(Ho**2)*(Tv**3))/(1 + Ho*Tv)

Tv_m = Tv_a/1000000       # Light travel time, in millions of ly
Do_m = (Do/(9.4693e18))   # Do_m = The initial distances, in millions
                  # of light years.

print("Initial distance in millions of light years:")
print(Do_m)

%matplotlib inline
```

```
import matplotlib.pyplot as plt

plt.figure(figsize = (10,4))
plt.plot(Tv_m, Do_m,'r-')
plt.legend(["Initial distance as a function of travel time."])
plt.xlabel('Distance traveled by light, Dv, million light years.', fontsize=12)
plt.ylabel('Initial distance, Do, million light years.', fontsize=12)
plt.title("The initial distance of the galaxies is calculated,\n based on the time
elapsed for the light to be detected.")
plt.savefig('Do(Dv).png')


# 3. CALCULATION OF THE DISTANCE THAT THE GALAXIES MOVED, DM, AS A
FUNCTION OF
# THE DISTANCE TRAVELED BY LIGHT OR THE TRAVEL TIME OF LIGHT.

print("Da_m = ")
print(Da_m)
print()
print("Do_m = ")
print(Do_m)
print()
print("Tv_m =")
print(Tv_m)
print()

Dm = (1/2)*(Da_m - Do_m)

print("Dm = ")
print(Dm)

plt.figure(figsize = (10,5))
plt.plot(Tv_m, Dm,'r--',linewidth=3)
plt.legend(["Movement or offset distance, Dm(Tv_m)."])
plt.xlabel('Travel time, Tv_m, in millions of years.', fontsize=12)
plt.ylabel('Distance moved, Dm, in millions of light years.', fontsize=12)

plt.xlim([-100, 14100])
plt.ylim([-100, 14100])
plt.grid(True)

plt.title("The distance of displacement or movement, Dm, since the light left
the sending galaxy, until it could be detected in the receiving galaxy.")
plt.savefig('Dm(tv).png')
```

```
# 4. CALCULATION OF THE COMMOVING DISTANCE, Dcm, AT WHICH
# YOU CAN FIND THE TRANSMITTER GALAXY OF THE RECEIVER,
# AS A FUNCTION OF THE SCALE FACTOR, a(t).

import numpy as np
import scipy as sp
%matplotlib inline
import matplotlib.pyplot as plt

Dv_al = np.linspace(500*pow(10,6),13500*pow(10,6),14)
      # Dv_al = Distance traveled in light years.

Dv_km = (Dv_al)*(9.4693*pow(10,12))
      # Dv_km = Distance traveled in km. (al)*(km/al) = km
Ho = (71.0)/(3.087*pow(10,19))    # Hubble constant, (km/seg)/(Km) = sec-
1
print("Ho = ",round(Ho,22)," 1/sec")
print()

c = 3e5

a_t = 0.6
Dcm1 = (1/a_t)*(Dv_km + (Ho/c)*(Dv_km**2))
Da_al1 = Dcm1/(9.4693*pow(10,12))     # (km)/(km/al) = ly
Da_m1 = Da_al1/1000000            # Da_m1 = distance traveled by light
# in millions of light years.
a_t = 0.8
Dcm2 = (1/a_t)*(Dv_km + (Ho/c)*(Dv_km**2))
Da_al2 = Dcm2/(9.4693*pow(10,12))
Da_m2 = Da_al2/1000000

a_t = 1.0
Dcm3 = (1/a_t)*(Dv_km + (Ho/c)*(Dv_km**2))
Da_al3 = Dcm3/(9.4693*pow(10,12))
Da_m3 = Da_al3/1000000

print("Distances in millions of ly, for a(t) = 1.")
print(Da_m3)
print()

a_t = 1.2
Dcm4 = (1/a_t)*(Dv_km + (Ho/c)*(Dv_km**2))
Da_al4 = Dcm4/(9.4693*pow(10,12))
Da_m4 = Da_al4/1000000
```

```
Dv_m = Dv_al/1000000

print("Distance traveled, Dv, in millions of ly.")
print(Dv_m)
print()

plt.figure(figsize = (8,5))
plt.plot(Dv_m,Da_m1,'b--',linewidth=2, label = 'a(t) = 0.6')
plt.plot(Dv_m,Da_m2,'r--',linewidth=2, label = 'a(t) = 0.8')
plt.plot(Dv_m,Da_m3,'k-' ,linewidth=2, label = 'a(t) = 1.0')
plt.plot(Dv_m,Da_m4,'g--',linewidth=2, label = 'a(t) = 1.2')

plt.legend(fontsize=12, loc=2)
plt.grid(True)

plt.xlabel('Distance traveled, Dv, millions of light years', fontsize=12)
plt.ylabel('Dcm, millions of light years ', fontsize=12)
plt.title("Comovil distance, Dcm, for various scale factors, a(t).", fontsize=12)
plt.savefig('Dcm(Dv_a(t)).png')
```