Original Research Article

Allocation of Subjects in An Educational Institution by Pascal Triangular Approach

ABSTRACT

The classical assignment problems cannot be successfully used for the real-world situation it is because the work efficiency varies to some extent from person to person hence, the use of fuzzy assignment problems is more appropriate. Based on the performance of the previous record in this study, on taking the scores obtained by the Chairman, by Course Director, by faculty members, and the student's feedback, we applied Pascal triangular approach in fuzzy assignment problem for allocating the subjects in a department of coming semester. A study of the technique for 4 jobs and 4 persons has been made.

Keywords: Pascal Triangular Approach, Fuzzy assignment, Triangular fuzzy number, Membership function.

INTRODUCTION

Effective teaching and learning are critically important to all students, especially those with special educational needs. In this, the allocation of subjects plays a vital role.

Andrew and Collins [1] proposed a mathematical model forassigning the subjects to teachersin the form of linear programming. The purpose of this model was to optimize the assignment of faculty member to courses subject to the number of courses needed and the faculty member load. Harwood and Lawless [8]used goal programming to examine the conflicting the goals in the faculty-course assignment problems. Aldy Gunawan, K. M. Ng and H. L. Ong [2] used Simulated Annealing and Tabu Search for the teacher assignment problem.

Assignment problems with fuzzy parameters have been studied by several authors such as Balinski [3] and Chi-Jen-Lin [4] and Chen [5], Kuhn Liu and Gao [6], Sathi, Mukherjee, and Kajla Basu [7].

Suppose there are ‘n’ people and ‘n’ jobs. Each job must be done by exactly one person; also, each person can do, at most, one job. The problem is to assign jobs to the people to minimize the total cost of completing all the jobs. The general assignment problem can be mathematically stated as follows: -

Minimize $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \quad for \;\; i = 1, 2, \ldots\ldots.n \qquad (\,one\ job\ is\ done\ by\ the\ i^{th}\ person,$$

$$i = 1, 2, \ldots\ldots n) \sum_{i=1}^{n} x_{ij} = 1 \quad for \;\; j$$

$$= 1, 2, \ldots\ldots n \quad (only\ one\ person\ should\ be\ assigned\ the\ \ j^{th}\ job,$$

$$j = 1, 2, \ldots n)$$

$$x_{ij} = \begin{cases} 1, & if\ i^{th}\ person\ assigned\ j^{th}\ job \\ 0, & if\ \ not \end{cases}$$

FUZZY ASSIGNMENT PROBLEM

Minimize $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} \widetilde{c_{ij}}\, x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \quad for \;\; i = 1,2, \ldots \ldots . n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad for \;\; j = 1,2, \ldots \ldots n$$

$x_{ij} = 0 \; or \; 1$.

TRIANGULAR FUZZY NUMBER

A fuzzy number A is a triangular fuzzy number denoted by (a_1, a_2, a_3) and its membership function $\mu_A(x)$ is given below:

$$\mu_A(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & a_1 \leq x \leq a_2 \\ 1 & x = a_2 \\ \frac{a_3 - x}{a_3 - a_2} & a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases}$$

CENTROID RANKING METHOD

The centroid of a triangle fuzzy number $\tilde{a} = (a, b, c; w)$ as $G_{\tilde{a}} = (\frac{a+b+c}{3}, \frac{w}{3})$. The ranking function of the generalized fuzzy number $\tilde{a} = (a, b, c; w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as $R(\tilde{a}) = (\frac{a+b+c}{3})(\frac{w}{3})$

ROBUST RANKING TECHNIQUE

Robust ranking technique which satisfies compensation, linearity, additive properties and provides results which are consists human intuition. If $\tilde{a}$ is a fuzzy number then the Robust ranking is defined by

$$R(\tilde{a}) = 0.5 \int_{0}^{1} (a_{\alpha}^{L}, a_{\alpha}^{U})\, d\alpha$$

where $(a_{\alpha}^{L}, a_{\alpha}^{U})$ is the α- level cut of the fuzzy number $\tilde{a}$ and

$$(a_{\alpha}^{L}, a_{\alpha}^{U}) = \{((b-a)+a), (d-(d-c))\}$$

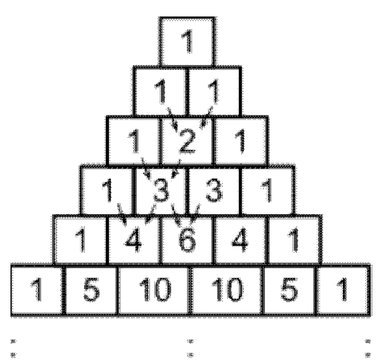
MAGNITUDE RANKING METHOD

For an arbitrary triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ with parametric form$\tilde{a} = (a(r), \bar{a}(r))$, the magnitude of the triangular fuzzy number $\tilde{a}$ by

$$Mag\,(\tilde{a}) = \frac{1}{2}\int_{0}^{1} (a_3 + 3a_1 - a_2)\, f(r)\, dr$$

Where the function $f(r)$ is a non- negative and increasing function on $[0,1]$. In the real-life applications $f(r)$ can be chosen by the decision maker according to the situation.

PASCAL TRIANGULAR APPROACH



Chen and Hrich proposed graded mean integration representation for representing generalized fuzzy number. This method is simply taken from the Pascal's triangles. These are useful to take the coefficient of fuzzy variables as Pascal triangular numbers and we just add and divided by the total Pascal numbers.

The graded mean approach for triangular fuzzy numbers(a, b, c).

$$P(A) = \frac{a + 2b + c}{4}$$

METHODOLOGY

Let there are four faculty members A, B, C, D and we have to assign four subjects I, II, III, IVto each of them. Each faculty member has obtained the scores by the Chairman, Course Director, and the department's students based on the previous semester's performance.

	I	II	III	IV
A	(1, 4, 7)	(3, 6, 9)	(7, 10, 13)	(2, 5, 8)
B	(8, 11, 14)	(5, 8, 11)	(4, 7, 10)	(6, 9, 12)
C	(9, 12, 15)	(0, 3, 6)	(1, 4, 7)	(4, 7, 10)
D	(7, 10, 13)	(8, 11, 14)	(2, 5, 8)	(3, 6, 9)

By Pascal triangular approach

$\boldsymbol{P(1, 4, 7)}$The membership function of the triangular fuzzy number$(1, 4, 7)$ is

$$\mu(x) = \begin{cases} \frac{x-1}{3}, & 1 \le x \le 7 \\ 1, & x = 4 \\ \frac{7-x}{3}, & 4 \le x \le 7 \\ 0, & otherwise. \end{cases}$$

$$P(1,4,7) = \frac{1 + 2(4) + 7}{4} = \frac{16}{4} = 4$$

$\boldsymbol{P(3,6,9)}$The membership function of the triangular fuzzy number $(3,6,9)$ is

$$\mu(x) = \begin{cases} \frac{x-3}{3}, & 3 \le x \le 6 \\ 1, & x = 6 \\ \frac{9-x}{3}, & 6 \le x \le 9 \\ 0, & otherwise. \end{cases}$$

$$P(3,6,9) = \frac{3 + 2(6) + 9}{4} = \frac{24}{4} = 6$$

$\boldsymbol{P(7,10,13)}$The membership function of the triangular fuzzy number $(7,10,13)$ is

$$\mu(x) = \begin{cases} \frac{x-7}{3}, & 7 \leq x \leq 10 \\ 1, & x = 10 \\ \frac{13-x}{3}, & 10 \leq x \leq 13 \\ 0, & otherwise. \end{cases}$$

$$P(7,10,13) = \frac{7 + 2(10) + 13}{4} = \frac{40}{4} = 10$$

$\boldsymbol{P(2,5,8)}$The membership function of the triangular fuzzy number $(2,5,8)$ is

$$\mu(x) = \begin{cases} \frac{x-2}{3}, & 2 \leq x \leq 5 \\ 1, & x = 5 \\ \frac{8-x}{3}, & 5 \leq x \leq 8 \\ 0, & otherwise. \end{cases}$$

$$P(2,5,8) = \frac{2 + 2(5) + 8}{4} = \frac{20}{4} = 5$$

$\boldsymbol{P(8,11,14)}$The membership function of the triangular fuzzy number $(8,11,14)$ is

$$\mu(x) = \begin{cases} \frac{x-8}{3}, & 8 \leq x \leq 11 \\ 1, & x = 11 \end{cases}$$

$\frac{14-x}{3}, \quad 11 \le x \le 14$

$0, \quad otherwise.$

$$P\,(8,11,14) = \frac{8+2(11)+14}{4} = \frac{44}{4} = 11$$

$\boldsymbol{P(5,8,11)}$The membership function of the triangular fuzzy number $(5,8,11)$ is

$$\mu(x) = \begin{cases} \frac{x-5}{3}, & 5 \le x \le 8 \\ 1, & x = 8 \\ \frac{11-x}{3}, & 8 \le x \le 11 \\ 0, & otherwise. \end{cases}$$

$$P\,(5,8,11) = \frac{5+2(8)+11}{4} = \frac{32}{4} = 8$$

$\boldsymbol{P(4,7,10)}$

The membership function of the triangular fuzzy number $(4,7,10)$ is

$$\mu(x) = \begin{cases} \frac{x-4}{3}, & 4 \le x \le 7 \\ 1, & x = 7 \\ \frac{10-x}{3}, & 7 \le x \le 10 \\ 0, & otherwise. \end{cases}$$

$$P\,(4,7,10) = \frac{4+2(7)+10}{4} = \frac{28}{4} = 7$$

$$P(6, 9, 12)$$

The membership function of the triangular fuzzy number $(6, 9, 12)$ is

$$\mu(x) = \begin{cases} \frac{x-6}{3}, & 6 \leq x \leq 9 \\ 1, & x = 9 \\ \frac{12-x}{3}, & 9 \leq x \leq 12 \\ 0, & otherwise. \end{cases}$$

$$P(6, 9, 12) = \frac{6 + 2(9) + 12}{4} = \frac{36}{4} = 9$$

$$P(9, 12, 15)$$

The membership function of the triangular fuzzy number $(9, 12, 15)$ is

$$\mu(x) = \begin{cases} \frac{x-9}{3}, & 9 \leq x \leq 12 \\ 1, & x = 12 \\ \frac{15-x}{3}, & 12 \leq x \leq 15 \\ 0, & otherwise. \end{cases}$$

$$P(9, 12, 15) = \frac{9 + 2(12) + 15}{4} = \frac{48}{4} = 12$$

$\boldsymbol{P(0,3,6)}$The membership function of the triangular fuzzy number $(0,3,6)$ is

$$\mu(x) = \begin{cases} \frac{x-0}{3}, & 0 \le x \le 3 \\ 1, & x = 3 \\ \frac{6-x}{3}, & 3 \le x \le 6 \\ 0, & otherwise. \end{cases}$$

$$P\,(0,3,6) = \frac{0 + 2(3) + 6}{4} = \frac{12}{4} = 3$$

After putting these values, we get the assignment problem

	I	II	III	IV
A	4	6	10	5
B	11	8	7	9
C	12	3	4	7
D	10	11	5	6

By Hungarian method, the optimal assignment is as follows:

Negate all values

Because the objective is to maximize the total cost, we negate all elements:

-4	-6	-10	-5
-11	-8	-7	-9
-12	-3	-4	-7
-10	-11	-5	-6

Make the matrix nonnegative

The cost matrix contains negative elements, we add 12 to each entry to make the cost matrix nonnegative:

8	6	2	7
1	4	5	3
0	9	8	5
2	1	7	6

Subtract row minima

We subtract the row minimum from each row:

6	4	0	5	(-2)
0	3	4	2	(-1)
0	9	8	5	
1	0	6	5	(-1)

Subtract column minima

We subtract the column minimum from each column:

6	4	0	3
0	3	4	0
0	9	8	3
1	0	6	3

			(-2)

Cover all zeros with a minimum number of lines

There are 4 lines required to cover all zeros:

6	**4**	**0**	**3**	***x***
0	**3**	**4**	**0**	***x***
0	**9**	**8**	**3**	***x***
1	**0**	**6**	**3**	***x***

The optimal assignment

Because there are 4 lines required, the zeros cover an optimal assignment:

6	4	**0**	3
0	3	4	**0**
0	9	8	3

1	**0**	6	3

This corresponds to the following optimal assignment in the original cost matrix:

4	6	**10**	5
11	8	7	**9**
12	3	4	7
10	**11**	5	6

The optimal assignment of the subjects is:

$A \rightarrow III$

$B \rightarrow IV$

$C \rightarrow I$

$D \rightarrow II$

The optimum value is 42.

CONCLUSION

In an educational institution, allocation of subjects to the faculty members plays an important role. In this paper, we apply Pascal triangular approach, to allocate the subjects to the faculty members. Here, we have used the scores given to the faculty members by the Chairman, Course Director, and the department's s in the previous semester. I have applied Robust ranking method in the previous paper .In Robust ranking method and magnitude ranking method, we have to integrate that may be complex or tedious. But Pascal's triangular approach is easier to apply.Here, we take the coefficients of fuzzy variables and then add and divided by total Pascal. We can apply this method to different sectors like health, human resource etc.

REFERENCES

1. Andrew, G. M & Collins, R" Matching faculty to course, College and University", Vol. 46, No 2, pp. 83-89, 1971.
2. Gunawan Aldy, Ng. K. M, Ong. H. L," Solving the Teacher Assignment Problem Two Metaheuristics", International Journal of Information and Management Sciences, Vol. 22, pp.73-86, 2011
3. Balinski, M. L "A Competitive simplex method for the assignment problem", A Math Program, 34(2), 125-141, (1986)

4. Chi-Jen Lin and Ue-pyng Wen, "A labeling algorithm for the fuzzy assignment problem", Fuzzy Sets & Systems 142, 2004
5. Chen M. S, "On a fuzzy assignment problem", Tamkang Journal of Mathematics, 22, pp. 407 – 411, (1985)
6. Kuhn H. W, "The Hungarian method for assignment problems", Naval Research Logistics Quarterly, 2, 83 – 97 (1955)
7. Mukherjee S. and Basu K, "Applications of a fuzzy ranking method for solving assignment problems with fuzzy costs", International Journal of Computational & Applied Mathematics 5, pp. 359 – 368 (2010)
8. Harwood, G. B, and Lawless, R. W, "Optimizing organizational goals in assigning faculty teaching schedules", Decisions Sciences, Vol. 6, pp. 513 – 524, 1975
9. Prabhakaran, K, and Ganesan K, "Fuzzy Hungarian Method for solving Intuitionistic Fuzzy Assignment problem", International Journal of Scientific & Engineering Research, Volume 6, Issue 3, March-2015, ISSN 2229-5518
10. Muruganandam, S and Hema, K, "Solving Fuzzy Assignment Problem using Fourier Elimination method", Global Journal of Pure & Applied Mathematics, Vol. 13, Number 2 (2017), pp 453 – 462, ISSN 0973 – 1768
11. Regina, Y. Liu and Joseph W, "Robust Rank – Based and Nonparametric Methods", Springer, April 2015
12. Ramesh Kumar, M and Subramanian, "Solution of Fuzzy Transportation Problems with Trapezoidal Fuzzy Numbers using Robust Methodology", International Journal of Pure and Applied Mathematics, Vol. 119 No. 16 2018, 3763 – 3775

13. Queen Mary, R and Selvi, D, “Solving Fuzzy Assignment problem using centroid Ranking Method”, International Journal of Mathematics and its Applications, 6 (3) (2018), 9-16

14. Selvi, D. Queen Mary, R and studentsVelammal, G. “Method for Solving fuzzy assignment problems using Magnitude Ranking Technique”, International Journal of Applied & Advanced Scientific Research, Special issue, Feb 2017, pp. 16 – 20

15. S. Subasri and K. Selvakumari, “Solving Neutrosophic Travelling Salesman Problem in Triangular Fuzzy Number using One Assignment Method”, Eurasian Journal of Analytical Chemistry. ISSN: 1306-3057 OPEN ACCESS 2019 14(1), 84-90.

16. Arun Pratap Singh, “Allocation of Subjects in an Educational Institution by Robust Ranking Method”, Aryabhatta Journal of Mathematics & Informatics, Vol. 13, No. 2, Jul-Dec., 2021, ISSN: 2394-9309.