

# Original Research Article

## **Allocation of Subjects in An Educational Institution by Pascal Triangular Approach**

### **ABSTRACT**

The classical assignment problems cannot be successfully used for the real-world situation it is because the work efficiency varies to some extent from person to person hence, the use of fuzzy assignment problems is more appropriate. Based on the performance of the previous record in this study, on taking the scores obtained by the Chairman, by Course Director, by faculty members, and the student's feedback, we applied Pascal triangular approach in fuzzy assignment problem for allocating the subjects in a department of coming semester. A study of the technique for 4 jobs and 4 persons has been made.

**Keywords:** Pascal Triangular Approach, Fuzzy assignment, Triangular fuzzy number, Membership function.

### **INTRODUCTION**

Effective teaching and learning are critically important to all students, especially those with special educational needs. In this, the allocation of subjects plays a vital role.

Andrew and Collins [1] developed a procedure for assigning the subjects to the teachers. Hawood and Lawless [8] used goal programming to solve the teacher assignment problem. Aldy Gunawan, K. M. Ng and H. L. Ong [2] used a genetic algorithm for the teacher assignment problem.

Assignment problems with fuzzy parameters have been studied by several authors such as Balinski [3] and Chi-Jen-Lin [4] and Chen [5], Kuhn Liu and Gao [6], Sathi, Mukherjee, and Kajla Basu [7].

Suppose there are 'n' people and 'n' jobs. Each job must be done by exactly one person; also, each person can do, at most, one job. The problem is to assign jobs to the people to minimize the total cost of completing all the jobs. The general assignment problem can be mathematically stated as follows: -

$$\text{Minimize } Z = \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^n x_{ij} = 1 \text{ for } i = 1, 2, \dots, n \quad (\text{one job is done by the } i^{\text{th}} \text{ person, } i = 1, 2, \dots, n)$$

$$\sum_{i=1}^n x_{ij} = 1 \text{ for } j = 1, 2, \dots, n \quad (\text{only one person should be assigned the } j^{\text{th}} \text{ job, } j = 1, 2, \dots, n)$$

$$x_{ij} = \begin{cases} 1, & \text{if } i^{\text{th}} \text{ person assigned } j^{\text{th}} \text{ job} \\ 0, & \text{if not} \end{cases}$$

### FUZZY ASSIGNMENT PROBLEM

$$\text{Minimize } Z = \sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^n x_{ij} = 1 \text{ for } i = 1, 2, \dots, n$$

$$\sum_{i=1}^n x_{ij} = 1 \text{ for } j = 1, 2, \dots, n$$

$$x_{ij} = 0 \text{ or } 1.$$

## TRIANGULAR FUZZY NUMBER

A fuzzy number A is a triangular fuzzy number denoted by  $(a_1, a_2, a_3)$  and its membership function  $\mu_A(x)$  is given below:

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ 1 & x = a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

## CENTROID RANKING METHOD

The centroid of a triangle fuzzy number  $\tilde{a} = (a, b, c; w)$  as  $G_{\tilde{a}} = (\frac{a+b+c}{3}, \frac{w}{3})$ . The ranking function of the generalized fuzzy number  $\tilde{a} = (a, b, c; w)$  which maps the set of all fuzzy numbers to a set of real numbers is defined as  $R(\tilde{a}) = (\frac{a+b+c}{3})(\frac{w}{3})$

## ROBUST RANKING TECHNIQUE

Robust ranking technique which satisfies compensation, linearity, additive properties and provides results which are consists human intuition. If  $\tilde{a}$  is a fuzzy number then the Robust ranking is defined by

$$R(\tilde{a}) = 0.5 \int_0^1 (a_{\alpha}^L, a_{\alpha}^U) d\alpha$$

where  $(a_{\alpha}^L, a_{\alpha}^U)$  is the  $\alpha$ - level cut of the fuzzy number  $\tilde{a}$  and

$$(a_{\alpha}^L, a_{\alpha}^U) = \{((b-a) + a), (d - (d-c))\}$$

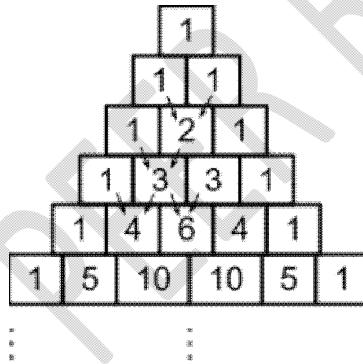
## MAGNITUDE RANKING METHOD

For an arbitrary triangular fuzzy number  $\tilde{a} = (a_1, a_2, a_3)$  with parametric form  $\tilde{a} = (a(r), \bar{a}(r))$ , the magnitude of the triangular fuzzy number  $\tilde{a}$  by

$$Mag(\tilde{a}) = \frac{1}{2} \int_0^1 (a_3 + 3a_1 - a_2) f(r) dr$$

Where the function  $f(r)$  is a non- negative and increasing function on  $[0,1]$ . In the real-life applications  $f(r)$  can be chosen by the decision maker according to the situation.

## PASCAL TRIANGULAR APPROACH



Chen and Hrich proposed graded mean integration representation for representing generalized fuzzy number. This method is simply taken from the Pascal's triangles. These are useful to take the coefficient of fuzzy variables as Pascal triangular numbers and we just add and divided by the total Pascal numbers.

The graded mean approach for triangular fuzzy numbers  $(a, b, c)$ .

$$P(A) = \frac{a + 2b + c}{4}$$

## METHODOLOGY

Let there are four faculty members A, B, C, D and we have to assign four subjects I, II, III, IV to each of them. Each faculty member has obtained the scores by the Chairman, Course Director, and the department's students based on the previous semester's performance.

	I	II	III	IV
A	(1, 4, 7)	(3, 6, 9)	(7, 10, 13)	(2, 5, 8)
B	(8, 11, 14)	(5, 8, 11)	(4, 7, 10)	(6, 9, 12)
C	(9, 12, 15)	(0, 3, 6)	(1, 4, 7)	(4, 7, 10)
D	(7, 10, 13)	(8, 11, 14)	(2, 5, 8)	(3, 6, 9)

### By Pascal triangular approach

$P(1, 4, 7)$  The membership function of the triangular fuzzy number  $(1, 4, 7)$  is

$$\mu(x) = \begin{cases} \frac{x-1}{3}, & 1 \leq x \leq 4 \\ 1, & x = 4 \\ \frac{7-x}{3}, & 4 \leq x \leq 7 \\ 0, & \text{otherwise.} \end{cases}$$

$$P(1, 4, 7) = \frac{1 + 2(4) + 7}{4} = \frac{16}{4} = 4$$

**P(3, 6, 9)** The membership function of the triangular fuzzy number (3, 6, 9) is

$$\mu(x) = \begin{cases} \frac{x-3}{3}, & 3 \leq x \leq 6 \\ 1, & x = 6 \\ \frac{9-x}{3}, & 6 \leq x \leq 9 \\ 0, & \text{otherwise.} \end{cases}$$

$$P(3, 6, 9) = \frac{3 + 2(6) + 9}{4} = \frac{24}{4} = 6$$

**P(7, 10, 13)** The membership function of the triangular fuzzy number (7, 10, 13) is

$$\mu(x) = \begin{cases} \frac{x-7}{3}, & 7 \leq x \leq 10 \\ 1, & x = 10 \\ \frac{13-x}{3}, & 10 \leq x \leq 13 \\ 0, & \text{otherwise.} \end{cases}$$

$$P(7, 10, 13) = \frac{7 + 2(10) + 13}{4} = \frac{40}{4} = 10$$

**P(2, 5, 8)** The membership function of the triangular fuzzy number (2, 5, 8) is

$$\mu(x) = \begin{cases} \frac{x-2}{3}, & 2 \leq x \leq 5 \\ 1, & x = 5 \\ \frac{8-x}{3}, & 5 \leq x \leq 8 \\ 0, & \text{otherwise.} \end{cases}$$

$$P(2, 5, 8) = \frac{2 + 2(5) + 8}{4} = \frac{20}{4} = 5$$

**P(8, 11, 14)** The membership function of the triangular fuzzy number (8, 11, 14) is

$$\mu(x) = \begin{cases} \frac{x-8}{3}, & 8 \leq x \leq 11 \\ 1, & x = 11 \\ \frac{14-x}{3}, & 11 \leq x \leq 14 \\ 0, & \text{otherwise.} \end{cases}$$

$$P(8, 11, 14) = \frac{8 + 2(11) + 14}{4} = \frac{44}{4} = 11$$

**P(5, 8, 11)** The membership function of the triangular fuzzy number (5, 8, 11) is

$$\mu(x) = \begin{cases} \frac{x-5}{3}, & 5 \leq x \leq 8 \\ 1, & x = 8 \\ \frac{11-x}{3}, & 8 \leq x \leq 11 \\ 0, & \text{otherwise.} \end{cases}$$

$$P(5, 8, 11) = \frac{5 + 2(8) + 11}{4} = \frac{32}{4} = 8$$

**$P(4, 7, 10)$**

The membership function of the triangular fuzzy number (4, 7, 10) is

$$\mu(x) = \begin{cases} \frac{x-4}{3}, & 4 \leq x \leq 7 \\ 1, & x = 7 \\ \frac{10-x}{3}, & 7 \leq x \leq 10 \\ 0, & \text{otherwise.} \end{cases}$$

$$P(4, 7, 10) = \frac{4 + 2(7) + 10}{4} = \frac{28}{4} = 7$$

**$P(6, 9, 12)$**

The membership function of the triangular fuzzy number (6, 9, 12) is

$$\mu(x) = \begin{cases} \frac{x-6}{3}, & 6 \leq x \leq 9 \\ 1, & x = 9 \\ \frac{12-x}{3}, & 9 \leq x \leq 12 \\ 0, & \text{otherwise.} \end{cases}$$

$$P(6, 9, 12) = \frac{6 + 2(9) + 12}{4} = \frac{36}{4} = 9$$

**$P(9, 12, 15)$**

The membership function of the triangular fuzzy number (9, 12, 15) is

$$\mu(x) = \begin{cases} \frac{x-9}{3}, & 9 \leq x \leq 12 \\ 1, & x = 12 \\ \frac{15-x}{3}, & 12 \leq x \leq 15 \\ 0, & \text{otherwise.} \end{cases}$$

$$P(9, 12, 15) = \frac{9 + 2(12) + 15}{4} = \frac{48}{4} = 12$$

**$P(0, 3, 6)$**  The membership function of the triangular fuzzy number (0, 3, 6) is

$$\mu(x) = \begin{cases} \frac{x-0}{3}, & 0 \leq x \leq 3 \\ 1, & x = 3 \\ \frac{6-x}{3}, & 3 \leq x \leq 6 \\ 0, & \text{otherwise.} \end{cases}$$

$$P(0, 3, 6) = \frac{0 + 2(3) + 6}{4} = \frac{12}{4} = 3$$

After putting these values, we get the assignment problem

	I	II	III	IV
A	4	6	10	5
B	11	8	7	9
C	12	3	4	7
D	10	11	5	6

By Hungarian method, the optimal assignment is as follows:

**Negate all values**

Because the objective is to maximize the total cost, we negate all elements:

-4	-6	-10	-5
-11	-8	-7	-9
-12	-3	-4	-7
-10	-11	-5	-6

**Make the matrix nonnegative**

The cost matrix contains negative elements, we add 12 to each entry to make the cost matrix nonnegative:

8	6	2	7
1	4	5	3
0	9	8	5
2	1	7	6

**Subtract row minima**

We subtract the row minimum from each row:

6	4	0	5	(-2)
0	3	4	2	(-1)
0	9	8	5	
1	0	6	5	(-1)

### Subtract column minima

We subtract the column minimum from each column:

6	4	0	3
0	3	4	0
0	9	8	3
1	0	6	3
			(-2)

### Cover all zeros with a minimum number of lines

There are 4 lines required to cover all zeros:

<b>6</b>	<b>4</b>	<b>0</b>	<b>3</b>	<b>x</b>
<b>0</b>	<b>3</b>	<b>4</b>	<b>0</b>	<b>x</b>
<b>0</b>	<b>9</b>	<b>8</b>	<b>3</b>	<b>x</b>
<b>1</b>	<b>0</b>	<b>6</b>	<b>3</b>	<b>x</b>

## The optimal assignment

Because there are 4 lines required, the zeros cover an optimal assignment:

6	4	<b>0</b>	3
0	3	4	<b>0</b>
<b>0</b>	9	8	3
1	<b>0</b>	6	3

This corresponds to the following optimal assignment in the original cost matrix:

4	6	<b>10</b>	5
11	8	7	<b>9</b>
<b>12</b>	3	4	7
10	<b>11</b>	5	6

The optimal assignment of the subjects is:

$A \rightarrow III$

$B \rightarrow IV$

$C \rightarrow I$

$D \rightarrow II$

The optimum value is 42.

### CONCLUSION

In an educational institution, allocation of subjects to the faculty members plays an important role. In this paper, we apply Pascal triangular approach, to allocate the subjects to the faculty members. Here, we have used the scores given to the faculty members by the Chairman, Course Director, and the department's s in the previous semester. I have applied Robust ranking method in the previous paper and I applied Pascal's triangular approach. I found that Pascal's triangular approach is easier to apply. We can apply this method to different sectors like health, human resource etc.

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