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## Rings Domination Number of Some Mycielski Graphs

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### Abstract

A set  $S$  of a graph  $G = (V(G), E(G))$  is a rings dominating set if  $S$  is a dominating set and for every vertex in the complement of  $S$  has atleast two adjacent vertices. The caridinality of the minimum rings dominating set is the rings domination number of graph  $G$ , denoted by  $\gamma_{ri}(G)$ . In this paper we determine the exact rings domination number of the mycielski graphs of path graph, cycle graph, and crown graph including its parameter.

*Keywords: rings domination, rings domination number, Mycielski graph.*

## 1 Introduction

Domination is one of the fundamental concepts used as an active tool in graph theory. The initiation of a new parameter in domination in graphs called "rings domination number" has been of great interest among mathematicians. It was Abed and Al-Harere in [11] who first presented the notion of rings domination which put a condition to a set  $V(G) - S$ . The rings dominating set of  $G$ , where  $G$  is a nontrivial connected graph with no isolated vertex, is defined as, a non-empty subset  $S \subseteq V(G)$  such that  $S$  is a dominating set and every vertex  $v \in V(G) - S$  has atleast two adjacent vertices in  $V(G) - S$ .

On one hand, Jan Mycielski in 1955 showed that there exist a triangle-free graphs with arbitrarily large chromatic number. Mycielski [4] introduced the graph-transformations as follows. Let  $G$  be a traingle-free graph. For each  $v_i \in V(G)$ , add a vertex  $v'_i$  to  $U$  adjacent to the same vertices of  $G$  that  $v_i$  is adjacent to. Finally, add a vertex  $w$  adjacent to each  $v'_i$  [8].

In this paper, we investigate the rings domination number of the mycielski graph of path graphs, cycle graphs, and crown graphs. For basic graph theory terminologies not specifically defined in this paper, please refer to either [5] or [11].

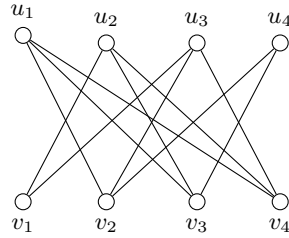
## 2 Preliminary Notes

This section contains some of the fundamental concepts necessary for the understanding of the study.

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**Definition 2.1.** [9] (**Crown Graph**) A crown graph  $G(n, n)$  is a graph on  $2n$  vertices with two sets of vertices  $u_i$  and  $v_j$  and with an edge from  $u_i$  to  $v_j$  whenever  $i \neq j$ .

**Example 2.1.** The graph in Figure 1 can be partitioned into two partite sets,  $U$  and  $V$ . Pick the vertex  $u_1$ . Then,  $u_1$  is adjacent to the vertices  $v_2, v_3,$  and  $v_4$ , except to the vertex  $v_1$ . Hence, it is therefore equivalent to the complete bipartite graph  $K_{(n,n)}$  without the horizontal edges.



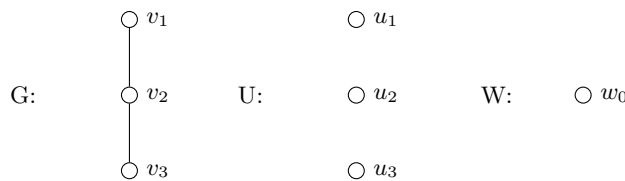
**Figure 1:** The crown graph of  $G(4, 4)$ .

**Definition 2.2.** [6] (**Mycielski Graph**) Consider a graph  $G$  with  $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$ . Apply the following steps to the graph  $G$ :

- (i) Take the set of new vertices  $U = \{u_1, u_2, u_3, \dots, u_n\}$  and add edges from each vertex  $u_i$  of  $U$  to the vertices  $v_j$  if the corresponding vertex  $v_i$  is adjacent to  $v_j$  in  $G$ .
- (ii) Take another new vertex  $w_0$  and add edges joining each element in  $U$ .

Here, the new graph obtained is the Mycielski graph of graph  $G$ .

**Example 2.2.** The graph  $G$  in Figure 2 is a path graph  $P_3$ . By (i), we take a new vertex set  $U$  that has the same cardinality of  $G$ . By (ii), we add a new vertex,  $w_0$ .



**Figure 2:** The Mycielski construction of path graph  $P_3$ .

By definition 2.2, we obtained the Mycielski graph of path graph  $\mu(P_3)$  illustrated in Figure 2 below. Observe that we add the edges from each vertex  $u_i \in U$  to the vertices  $v_j$  to the corresponding vertex  $v_i$  adjacent to  $v_j$  in  $G$ . Finally, we add an edge joining each  $u_i \in U$  and the vertex  $w_0$ .

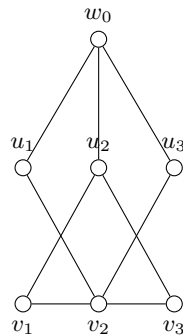


Figure 3: The Mycielski graph of  $\mu(P_3)$ .

**Definition 2.3.** [11] (**Rings Dominating Set, Rings Domination Number**) Let  $G = (V(G), E(G))$  be a nontrivial connected graph with no isolated vertex. A non-empty set  $S \subseteq V(G)$  is rings dominating set of graph  $G$  if,  $S$  is a dominating set and for every  $v \in V(G) - S$  has atleast two adjacent vertices in  $V(G) - S$ . The cardinality of a minimum rings dominating set of  $G$  is the rings domination number of  $G$ , denoted by  $\gamma_{ri}(G)$ . A set  $S_0 \subseteq V(G)$ , where  $|S_0| = \gamma_{ri}(G)$  is called the  $\gamma_{ri}$ -set of  $G$ .

**Example 2.3.** Consider the graph  $G$  in Figure 4 below. Take the set  $S = \{u_{13}\}$ . Clearly, the set  $S$  is a dominating set. Moreover, every vertex in  $V(G) - S$ ,  $deg(u_1) = deg(u_2) = deg(u_3) = deg(u_4) = 4$  and  $deg(u_5) = deg(u_6) = deg(u_7) = deg(u_8) = deg(u_9) = deg(u_{10}) = deg(u_{11}) = deg(u_{12}) = 2$ , has atleast two adjacent vertices in  $V(G) - S$ . Obviously,  $S$  is a minimum rings dominating set of graph  $G$ . Hence, the rings domination number of  $G$  is  $\gamma_{ri}(G) = 1$ .

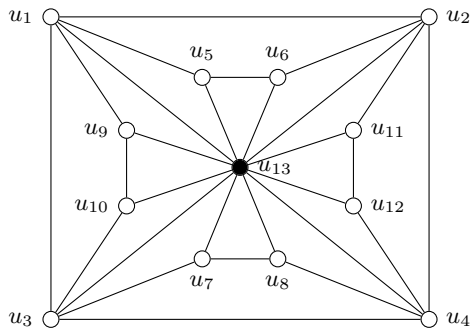


Figure 4: The graph  $G$

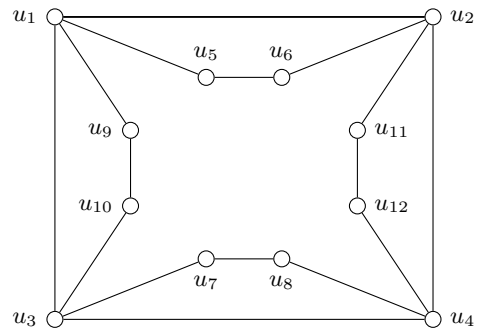


Figure 5: The subgraph  $S$  of graph  $G$

### 3 Main Results

In this section, the rings domination number of Mycielski graph of path graph, cycle graph, and crown graph are shown. As well as, the parameter of a rings domination number of the mentioned graphs.

### 3.1 Rings Domination Number of the Mycielski Graph of Path Graph, $\mu(P_n)$

For convenience, we consider the path graph  $P_n$  of order  $n \geq 4$  with vertex set  $V(P_n) = \{v_1, v_2, \dots, v_n\}$ . The following shows the parameter of the rings domination number of the mycielski graph of path graph via modulus.

**Theorem 3.1.** *Let  $P_n$  be a path graph with  $n \geq 4$ . Then,*

$$\gamma_{ri}(\mu(P_n)) = \begin{cases} \frac{n+8}{2}, & \text{if } n \equiv 0(\text{mod } 4) \\ \frac{n+5}{2}, & \text{if } n \equiv 1(\text{mod } 4) \text{ or } n \equiv 3(\text{mod } 4) \\ \frac{n+6}{2}, & \text{if } n \equiv 2(\text{mod } 4) \end{cases}$$

*Proof:* Let  $V(\mu(P_n)) = V(P_n) \cup U \cup W$ , where  $V(P_n) = \{v_1, v_2, \dots, v_n\}$ ,  $U = \{u_1, u_2, \dots, u_n\}$ , and  $W = \{w_0\}$ . Suppose  $S$  is a  $\gamma_{ri}$ -set. Consider the following cases.

Case 1:  $n \equiv 0 \pmod{4}$

Choose  $R = \{v_1, v_5, v_9, \dots, v_{n-3}\} \cup \{v_4, v_8, \dots, v_n\} \cup \{u_1, u_n\}$ . Clearly,  $R$  is a rings dominating set. Since  $S$  is a  $\gamma_{ri}$ -set,  $|R| \geq |S|$ . Thus,  $|R| = \frac{n}{4} + 2 \geq |S|$ . On the other hand, since  $S$  is a  $\gamma_{ri}$ -set, then  $S$  must have atleast  $\frac{n}{4} + 2$  vertices in  $\mu(P_n)$ . Thus,  $|S| \geq \frac{n}{4} + 2$ . Therefore,  $\gamma_{ri}(\mu(P_n)) = |S| = \frac{n}{4} + 2 = \frac{n+8}{4}$ .

Case 2:  $n \equiv 1 \pmod{4}$  or  $n \equiv 3 \pmod{4}$

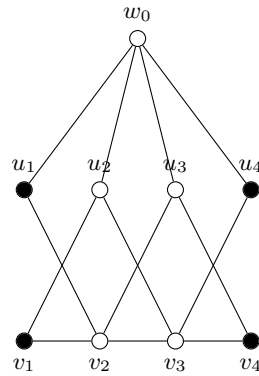
For  $n \equiv 1 \pmod{4}$ . Choose  $R = \{v_1, v_5, v_9, \dots, v_n\} \cup \{v_4, v_8, \dots, v_{n-1}\} \cup \{u_1, u_n\}$ . Clearly,  $R$  is a rings dominating set. Since  $S$  is a  $\gamma_{ri}$ -set,  $|R| \geq |S|$ . Thus,  $|R| = \frac{2n+2}{4} + 2 \geq |S|$ . On the other hand, since  $S$  is a  $\gamma_{ri}$ -set, then  $S$  must have atleast  $\frac{2n+2}{4} + 2$  vertices in  $\mu(P_n)$ . Thus,  $|S| \geq \frac{2n+2}{4} + 2$ . Therefore,  $\gamma_{ri}(\mu(P_n)) = |S| = \frac{2n+2}{4} + 2 = \frac{n+5}{2}$ .

Similarly for  $n \equiv 3 \pmod{4}$  by letting  $R = \{v_1, v_5, v_9, \dots, v_n\} \cup \{v_4, v_8, \dots, v_{n-3}\} \cup \{u_1, u_n\}$ .

Case 3:  $n \equiv 2 \pmod{4}$

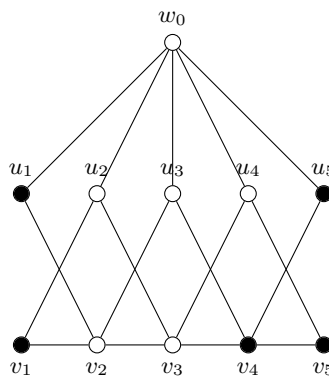
Choose  $R = \{v_1, v_5, v_9, \dots, v_{n-1}\} \cup \{v_2, v_6, \dots, v_n\} \cup \{u_1, u_n\}$ . Clearly,  $R$  is a rings dominating set. Since  $S$  is a  $\gamma_{ri}$ -set,  $|R| \geq |S|$ . Thus,  $|R| = \frac{2n+2}{4} + 2 \geq |S|$ . On the other hand, since  $S$  is a  $\gamma_{ri}$ -set, then  $S$  must have atleast  $\frac{2n+4}{4} + 2$  vertices in  $\mu(P_n)$ . Thus,  $|S| \geq \frac{2n+4}{4} + 2$ . Therefore,  $\gamma_{ri}(\mu(P_n)) = |S| = \frac{2n+4}{4} + 2 = \frac{n+6}{2}$ .  $\square$

**Example 3.2.** *Consider the Mycielski graph of path graph  $\mu(P_4)$ . It can be seen that if we let  $S \subseteq V(\mu(P_4))$ , where  $S = \{v_1, v_4, u_1, u_2\}$ , then clearly,  $S$  is a dominating set of the mycielski graph of  $\mu(P_4)$ . Moreover, every vertex in  $V(\mu(P_4)) - S = \{v_2, v_3, u_2, u_3, w_0\}$  has atleast two adjacent vertices in  $V(\mu(P_4)) - S$ . Observe that the set  $S$  has the smallest cardinality of all the rings dominating set in  $\mu(P_4)$ . Hence, the rings domination number of  $\mu(P_4)$  is  $\gamma_{ri}(\mu(P_4)) = 4$ .*



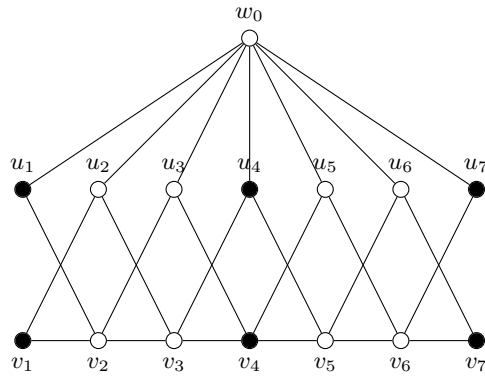
**Figure 6:** The Mycielski graph of  $\mu(P_4)$  and its rings dominating set  $S = \{v_1, v_4, u_1, u_2\}$ .

**Example 3.3.** Consider the Mycielski graph of path graph  $\mu(P_5)$ . It can be seen that if we let  $S \subseteq V(\mu(P_5))$ , where  $S = \{v_1, v_4, v_5, u_1, u_5\}$ , then clearly,  $S$  is a dominating set of the mycielski graph of  $\mu(P_5)$ . Moreover, every vertex in  $V(\mu(P_5)) - S = \{v_2, v_3, u_2, u_3, u_4, w_0\}$  has atleast two adjacent vertices in  $V(\mu(P_5)) - S$ . Observe that the set  $S$  has the smallest cardinality of all the rings dominating set in  $\mu(P_5)$ . Hence, the rings domination number of  $\mu(P_5)$  is  $\gamma_{ri}(\mu(P_5)) = 5$ .



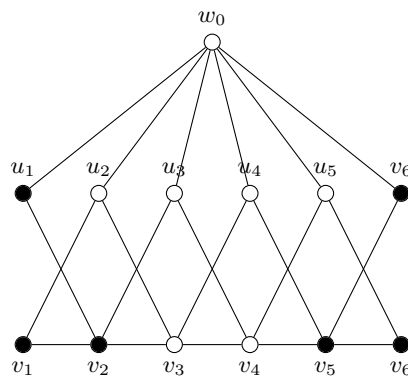
**Figure 7:** The Mycielski graph of  $\mu(P_5)$  and its rings dominating set  $S = \{v_1, v_4, v_5, u_1, u_5\}$ .

**Example 3.4.** Consider the Mycielski graph of path graph  $\mu(P_7)$ . Let  $S \subseteq V(\mu(P_7))$ , where  $S = \{v_1, v_4, v_7, u_1, u_4, u_7, w_0\}$ , then clearly,  $S$  is a dominating set of the mycielski graph of  $\mu(P_7)$ . Moreover, every vertex in  $V(\mu(P_7)) - S = \{v_2, v_3, v_5, v_6, u_2, u_3, u_4, u_5, u_6, w_0\}$  has atleast two adjacent vertices in  $V(\mu(P_7)) - S$ . Observe that the set  $S$  has the smallest cardinality of all the rings dominating set in  $\mu(P_7)$ . Hence, the rings domination number of  $\mu(P_7)$  is  $\gamma_{ri}(\mu(P_7)) = 6$ .



**Figure 8:** The Mycielski graph of  $\mu(P_7)$  and its rings dominating set  $S = \{v_1, v_4, v_7, u_1, u_4, u_7\}$ .

**Example 3.5.** Consider the Mycielski graph of path graph  $\mu(P_6)$ . It can be seen that if we let  $S \subseteq V(\mu(P_6))$ , where  $S = \{v_1, v_2, v_5, v_6, u_1, u_6\}$ , then clearly,  $S$  is a dominating set of the mycielski graph of  $\mu(P_6)$ . Moreover, every vertex in  $V(\mu(P_6)) - S = \{v_3, v_4, u_2, u_3, u_4, u_5, w_0\}$  has atleast two adjacent vertices in  $V(\mu(P_6)) - S$ . Observe that the set  $S$  has the smallest cardinality of all the rings dominating set in  $\mu(P_6)$ . Hence, the rings domination number of  $\mu(P_6)$  is  $\gamma_{ri}(\mu(P_6)) = 6$ .



**Figure 9:** The Mycielski Graph of  $\mu(P_6)$  and its rings dominating set  $S = \{v_1, v_2, v_5, v_6, u_1, u_6\}$ .

### 3.2 Rings Domination Number of the Mycielski Graph of Cycle Graph, $\mu(C_n)$

For convenience, we consider the cycle graph  $C_n$  of order  $n \geq 4$  with vertex set  $V(C_n) = \{v_1, v_2, \dots, v_n\}$ . The following shows the parameter of a rings domination number of the mycielski graph of cycle graph via modulus.

**Theorem 3.6.** Let  $C_n$  be a cycle graph with  $n \geq 4$ , then

$$\gamma_{ri}(\mu(C_n)) = \begin{cases} \frac{n+2}{2}, & \text{if } n \equiv 0(\text{mod } 4) \\ \frac{n+5}{2}, & \text{if } n \equiv 1(\text{mod } 4) \\ \frac{n+2}{2}, & \text{if } n \equiv 2(\text{mod } 4) \\ \frac{n+1}{2}, & \text{if } n \equiv 3(\text{mod } 4) \end{cases}$$

*Proof:* Let  $V(\mu(C_n)) = V(C_n) \cup U \cup W$ , where  $V(C_n) = \{v_1, v_2, \dots, v_n\}$ ,  $U = \{u_1, u_2, \dots, u_n\}$ , and  $W = \{w_0\}$ . Suppose  $S$  is a  $\gamma_{ri}$ -set. Consider the following cases.

Case 1:  $n \equiv 0 \pmod{4}$

Choose  $R = \{v_1, v_5, v_9, \dots, v_n\} \cup \{v_4, v_8, \dots, v_{n-3}\} \cup \{u_1\}$ . Clearly,  $R$  is a rings dominating set. Since  $S$  is a  $\gamma_{ri}$ -set,  $|R| \geq |S|$ . Thus,  $|R| = \frac{2n}{4} + 1 \geq |S|$ . On the other hand, since  $S$  is a  $\gamma_{ri}$ -set, then  $S$  must have atleast  $\frac{2n}{4} + 1$  vertices in  $\mu(C_n)$ . Thus,  $|S| \geq \frac{2n}{4} + 1$ . Therefore,  $\gamma_{ri}(\mu(C_n)) = |S| = \frac{2n}{4} + 1 = \frac{n+2}{2}$ .

Case 2:  $n \equiv 1 \pmod{4}$

Choose  $R = \{v_1, v_5, v_9, \dots, v_n\} \cup \{v_4, v_8, \dots, v_{n-1}\} \cup \{u_1, u_n\}$ . Clearly,  $R$  is a rings dominating set. Since  $S$  is a  $\gamma_{ri}$ -set,  $|R| \geq |S|$ . Thus,  $|R| = \frac{2n+2}{4} + 2 \geq |S|$ . On the other hand, since  $S$  is a  $\gamma_{ri}$ -set, then  $S$  must have atleast  $\frac{2n+2}{4} + 2$  vertices in  $\mu(C_n)$ . Thus,  $|S| \geq \frac{2n+2}{4} + 2$ . Therefore,  $\gamma_{ri}(\mu(C_n)) = |S| = \frac{2n+2}{4} + 2 = \frac{n+5}{2}$ .

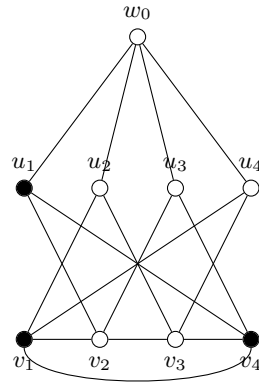
Case 3:  $n \equiv 2 \pmod{4}$

Choose  $R = \{v_1, v_5, v_9, \dots, v_{n-5}\} \cup \{v_4, v_8, \dots, v_{n-3}\} \cup \{u_1, u_{n-3}\}$ . Clearly,  $R$  is a rings dominating set. Since  $S$  is a  $\gamma_{ri}$ -set,  $|R| \geq |S|$ . Thus,  $|R| = \frac{2n-4}{4} + 2 \geq |S|$ . On the other hand, since  $S$  is a  $\gamma_{ri}$ -set, then  $S$  must have atleast  $\frac{2n-4}{4} + 2$  vertices in  $\mu(C_n)$ . Thus,  $|S| \geq \frac{2n-4}{4} + 2$ . Therefore,  $\gamma_{ri}(\mu(C_n)) = |S| = \frac{2n-4}{4} + 2 = \frac{n+2}{2}$ .

Case 4:  $n \equiv 3 \pmod{4}$

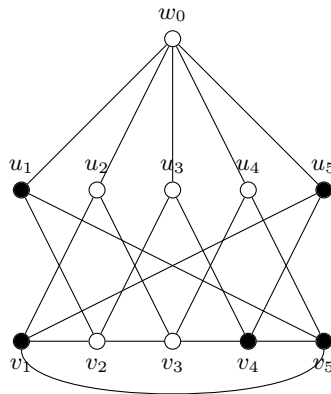
Choose  $R = \{v_1, v_5, v_9, \dots, v_{n-2}\} \cup \{v_4, v_8, \dots, v_{n-3}\} \cup \{u_1\}$ . Clearly,  $R$  is a rings dominating set. Since  $S$  is a  $\gamma_{ri}$ -set,  $|R| \geq |S|$ . Thus,  $|R| = \frac{2n-2}{4} + 1 \geq |S|$ . On the other hand, since  $S$  is a  $\gamma_{ri}$ -set, then  $S$  must have atleast  $\frac{2n-2}{4} + 1$  vertices in  $\mu(C_n)$ . Thus,  $|S| \geq \frac{2n-2}{4} + 1$ . Therefore,  $\gamma_{ri}(\mu(C_n)) = |S| = \frac{2n-2}{4} + 1 = \frac{n+1}{2}$ .  $\square$

**Example 3.7.** Consider the Mycielski graph of cycle graph  $\mu(C_4)$ . It can be seen that if we let  $S \subseteq V(\mu(C_4))$ , where  $S = \{v_1, v_4, u_1\}$ , then clearly,  $S$  is a dominating set of the mycielski graph of  $\mu(C_4)$ . Moreover, every vertex in  $V(\mu(C_4)) - S = \{v_2, v_3, u_2, u_3, u_4, w_0\}$  has atleast two adjacent vertices in  $V(\mu(C_4)) - S$ . Observe that the set  $S$  has the smallest cardinality of all the rings dominating set in  $\mu(C_4)$ . Hence, the rings domination number of  $\mu(C_4)$  is  $\gamma_{ri}(\mu(C_4)) = 3$ .



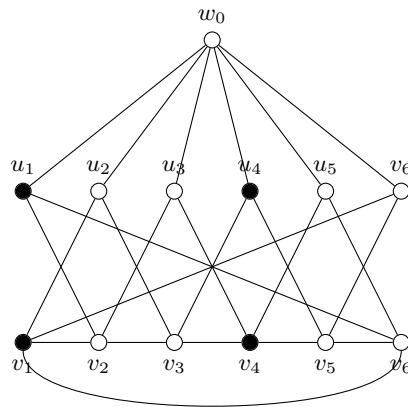
**Figure 10:** The Mycielski Graph of  $\mu(C_4)$  and its rings dominating set  $S = \{v_1, v_4, u_1\}$ .

**Example 3.8.** Consider the Mycielski graph of cycle graph  $\mu(C_5)$ . It can be seen that if we let  $S \subseteq V(\mu(C_5))$ , where  $S = \{v_1, v_4, v_5, u_1, u_5\}$ , then clearly,  $S$  is a dominating set of the mycielski graph of  $\mu(C_4)$ . Moreover, every vertex in  $V(\mu(C_5)) - S = \{v_2, v_3, u_2, u_3, u_4, w_0\}$  has atleast two adjacent vertices in  $V(\mu(C_5)) - S$ . Observe that the set  $S$  has the smallest cardinality of all the rings dominating set in  $\mu(C_5)$ . Hence, the rings domination number of  $\mu(C_5)$  is  $\gamma_{ri}(\mu(C_5)) = 5$ .



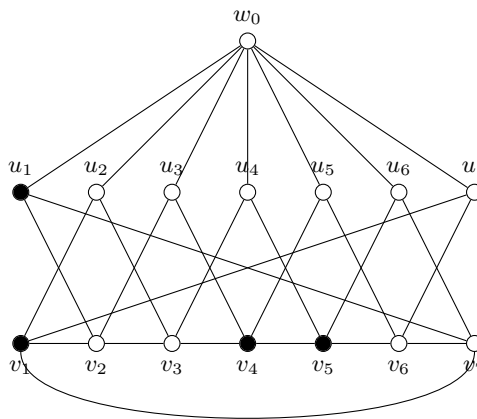
**Figure 11:** The Mycielski Graph of  $\mu(C_5)$  and its rings dominating set  $S = \{v_1, v_4, v_5, u_1, u_5\}$ .

**Example 3.9.** Consider the Mycielski graph of cycle graph  $\mu(C_6)$ . It can be seen that if we let  $S \subseteq V(\mu(C_6))$ , where  $S = \{v_1, v_4, u_1, u_4\}$ , then clearly,  $S$  is a dominating set of the mycielski graph of  $\mu(C_6)$ . Moreover, every vertex in  $V(\mu(C_6)) - S = \{v_2, v_3, v_5, v_6, u_2, u_3, u_5, u_6, w_0\}$  has atleast two adjacent vertices in  $V(\mu(C_6)) - S$ . Observe that the set  $S$  has the smallest cardinality of all the rings dominating set in  $\mu(C_6)$ . Hence, the rings domination number of  $\mu(C_6)$  is  $\gamma_{ri}(\mu(C_6)) = 4$ .



**Figure 12:** The Mycielski Graph of  $\mu(C_6)$  and its rings dominating set  $S = \{v_1, v_4, u_1, u_4\}$ .

**Example 3.10.** Consider the Mycielski graph of cycle graph  $\mu(C_7)$ . It can be seen that if we let  $S \subseteq V(\mu(C_7))$ , where  $S = \{v_1, v_4, v_5, u_1\}$ , then clearly,  $S$  is a dominating set of the mycielski graph of  $\mu(C_7)$ . Moreover, every vertex in  $V(\mu(C_7)) - S = \{v_2, v_3, v_6, v_7, u_2, u_3, u_4, u_5, u_6, u_7, w_0\}$  has atleast two adjacent vertices in  $V(\mu(C_7)) - S$ . Observe that the set  $S$  has the smallest cardinality of all the rings dominating set in  $\mu(C_7)$ . Hence, the rings domination number of  $\mu(C_7)$  is  $\gamma_{ri}(\mu(C_7)) = 4$ .



**Figure 13:** The Mycielski Graph of  $\mu(C_6)$  and its rings dominating set  $S = \{v_1, v_4, v_5, u_1\}$ .

### 3.3 Rings Domination Number of the Mycielski Graph of Crown Graph, $G(n, n)$

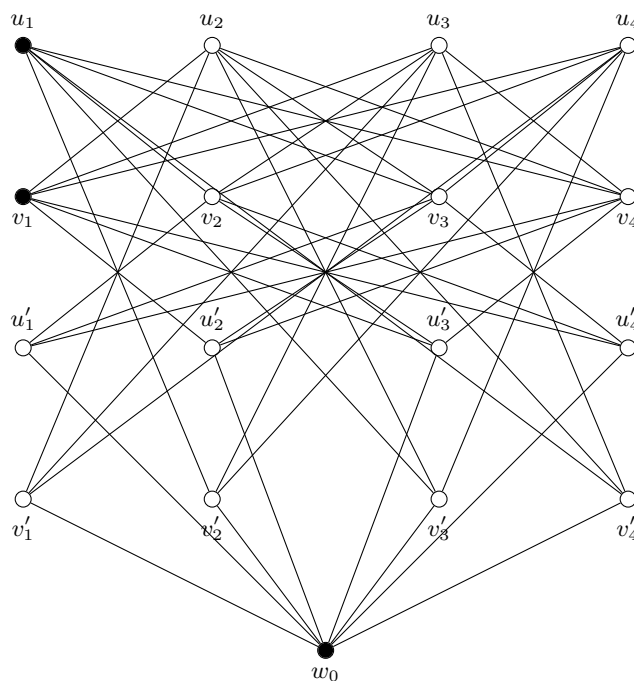
For convenience, we consider the crown graph  $G$  of order  $n \geq 4$ . The following shows the parameter of a rings domination number of the mycielski graph of crown graph.

**Proposition 3.1.** Let  $G(n, n)$  be a crown graph with  $n \geq 4$ . Then,  $\gamma_{ri}(\mu(G(n, n))) = 3$ .

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*Proof:* Let  $V(\mu(G(n, n))) = V(G(n, n)) \cup U \cup W$ , where  $V(G(n, n)) = \{u_1, u_2, \dots, u_n\} \cup \{v_1, v_2, \dots, v_n\}$ ,  $U = \{u'_1, u'_2, \dots, u'_n\} \cup \{v'_1, v'_2, \dots, v'_n\}$ , and  $W = \{w_0\}$ . Choose  $R = \{u_1, v_1, w_0\} \subseteq V(\mu(G(n, n)))$ . Clearly,  $R$  is a rings dominating set. Obviously, there can be no other rings dominating set smaller than  $R$ . Hence,  $\gamma_{ri}(\mu(G(n, n))) = |R| = 3$ .  $\square$

**Example 3.11.** Consider the Mycielski graph of crown graph  $\mu(G(4, 4))$ . It can be seen that if we let  $S \subseteq V(\mu(G(4, 4)))$ , where  $S = \{u_1, v_1, w_0\}$ , then clearly,  $S$  is a dominating set of the mycielski graph of  $\mu(G(4, 4))$ . Moreover, every vertex in  $V(\mu(G(4, 4))) - S$  has atleast two adjacent vertices in  $V(\mu(G(4, 4))) - S$ . Observe that the set  $S$  has the smallest cardinality of all the rings dominating set in  $\mu(G(4, 4))$ . Hence, the rings domination number of  $\mu(G(4, 4))$  is  $\gamma_{ri}(\mu(G(4, 4))) = 3$ .



**Figure 14:** The Mycielski graph of crown graph  $\mu(G(n, n))$  and its rings dominating set  $S = \{v_1, u_1, w_0\}$

## 4 Conclusion

In this article, the rings dominating set of the mycielski graph of path graph, cycle graph, and crown graph is observed. Furthermore, the parameter of the rings domination number of the mentioned graphs are determined.

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