

ORIGINAL RESEARCH ARTICLE

Universe before Big Bang

1. Abstract

The ghost condensation of the early universe in a pre-big bang phase has been presented in this paper through duration of a non-singular bounce. The undergoing universe contracts and passes smoothly in an expanding universe via a post-big bang phase. Initially developing and then taming any ghost like instabilities, the Null Energy Condition (NEC) is explicitly violated through the curvature mechanism of an adiabatic perturbed metric. The vacuum state of the ongoing phase is stabilized via a Lagrangian that in essence stabilizes the vacuum state under the higher order derivatives. The violation of the NEC regards a catastrophic vacuum instability, which re-emerges with a correction valid at small energies and momenta, below the UV-cut-off scale that, could potentially be problematic if one tries to construct a UV-completed theory of this Ekpyrotic model. The scale-invariant curvature perturbation, that arises and is sourced out of the scale-invariant entropy perturbations sourced by 2-Ekpyrotic scalar fields, that, in contrast, becomes constant on the super-horizon limits, due to the non-singular nature of the background geometry. Apart, from the ghost condensates, this theory addresses the new Ekpyrotic theory which in order becomes a distinguishable alternative to inflation theory for the birth of the universe. As per the recent WMAP data, the Ekpyrotic model has a spectral red tilt that shows the bounced scalar potential falling through a negative phase shift during the matter-fluid fluctuations in the hot big bang phase.

Keywords: GHOST Field – RADION Field – VACUUM Fluctuations – M-Theory – DIRICHLET(p)-Branes – RANDALL-SUNDRUM Model – ENTROPY & CURVATURE Perturbations – $N = 1$ Supergravity

2. Introduction

The temperature anisotropy of the Cosmic Microwave Background Radiation (CMBR) in the past decade has provided us the large-scale structure of the universe that originates from the primordial perturbations on a nearly Gaussian scale-invariant adiabatic scales [1]. As this coincides with the inflammatory models of the universe, therefore, it has been widely accepted as the evidence of inflammatory cosmology which holds a

breakthrough for solving different cosmological problems. However, there has been quite a research going on for an alternative view of the inflammatory cosmology and the new Ekpyrotic theory is one of them [2, 3]. To delve into the cosmological conclusions is a healthy approach if it succeeds; else, the additional demonstration has to make a way for the inflammatory cosmology. However, the original Ekpyrotic model as approached by different physicists failed to solve the problem of the large mass and entropy and along the way instead of solving the homogeneity problem solves the solution of the big crunch [4, 5] irrespective of the big bang. Then, with the phases of numerous expansion and contraction [6], the cyclic theory of the universe has been postulated that in turn that provides a better result than the originally proposed Ekpyrotic scenario, but it remains unclear about the origin of the scalar field potential while its implications have been visible. Afterward, when this potential has been analyzed along with the particle production of the early universe, a very different cosmological model comes to play [7, 8].

In this paper, a concrete scenario of the cyclic cosmology and that of the early universe has been presented having a scale-invariant spectrum of density fluctuations without taking the period of inflationary expansion [9, 10, 11]. The perturbations of the density fluctuations have been generated in a period before the big bang which provides a scale-invariant non-singular spectrum of density fluctuations of a 4-Dimensional effective field theory evolving from contraction to expansion and its effect on the hot matter-fluid phase of the big bang, the hot expanding phase [12].

The potential having a negative gradient which having a large exponent roll down the slope with scale-invariant fluctuations having 60_{-1}^{+1} wavenumbers in comoving scenario [13]. The inflammatory cosmology [14] shows us the exponential expansion with a nearly constant Hubble radius, but in this paper, with a decreasing or negative potential, through a rapidly decrement of Hubble radius explored via Friedmann-Lemaitre-Robertson-Walker (FLRW)^[see Appendix B] geometry. The dynamics of the inflammatory cosmology are different from the Ekpyrosis in a view that, in the former, the gravitational radiation is a scale invariant, while in the

latter, its amplitude has been exponentially suppressed [15] being invariant through the observational window.

Usually, in the cyclic phase, the NEC violation is an inert nature that leads to vacuum instability but, the authors [16, 17, 18, 19] argue that the instability occurs at the onset of the "bounce" from contracting to the expanding phase where it gives birth to tachyons or ghosts of arbitrarily large masses [20]. There occur Jeans instabilities near the bounce, and to remove these instabilities, a dampening of the higher derivative term $-(\square\phi)^2$ has been added with the ghost condensate [21]. However, this yields a problematic solution as the ghost condensate is UV-incomplete which indicates that the 2nd Type of theory is needed, the authors [22, 23, 24, 25] proposed the consecutive theories. Kallosh et al., 2008 [2] showed that the suppression of the perturbations by the damping of the higher derivative terms which instead leads to the creation of a new ghost in the new Ekpyrotic scenario. The density fluctuations (vacuum) of the scalar potential and the gauge-invariant Newtonian potential, being scale-invariant, turns out that, their growing fluctuation modes result in the canceling of the curvature perturbations $-\xi_0$ on uniform density hypersurfaces [26, 27]. A completely non-singular evolution has been considered and the curvature perturbations $-\xi_0$ is conserved. Here, 2 scalar potentials have been considered [1] which have near-exponentially negative potentials that is scale invariant of perturbations, and in doing so, the entropy perturbations get converted in the adiabatic mode which in case, makes the entropy perturbations, scale-invariant, long before the bounce thereby making a smooth transition from contraction to expansion in the big bang scenario [1]. The authors [1] having taken, the 2 scalar field potentials, in such a way that, 1st field enters the Ekpyrotic phase and exists the ghost condensate much earlier than the other giving enough time to make a sharp rise in the gradient-potentials which in the Ekpyrotic phase is negative while ghost condensate phase being positive.

To find the exact exponential potentials, the spectral tilt is blue while the WMAP gives a small red tilt which indicates that, there is a clear sign of deviation from the near exponential fields [28]. This is explicitly shown in the form of 2 scalar fields, albeit not exponentials [1]. This indicates, the hinge towards a near exponential red tilt.

It has been shown in [29, 30, 31] that, the equation of state could have a form of $\kappa \gg 1$ in the contracting phase, however, for the 4-dimensional gravity which ultimately breaks down before the bounce, the equation of state revert to $\kappa = 1$ that is subdominant to towards the singularity leads to a finite FLRW^[see Appendix B] background. This delves the conclusion that the perturbations get damped long before the singularity in a linear regime where the curvature coefficients of perturbations $-\xi_0$ goes through unscathed. The universe then smoothly started from a hot big bang phase with scale-invariant perturbations all the way ahead of time which is free from ghost instabilities. There should be a Dirichlet(p) – boundary wall of a 5-Dimensional brane in the bulk with a singularity free collision [29, 30, 31, 32].

The paper has been organized in the following ways: Section 3 contains a detailed explanation of the Ekpyrosis and its implications over the brane-world cosmology to the extent of its difference from the big bang which shows a definite distinction between singular and non-singular bounces. Section 4 consists of the two-field scalar potentials and their implications on Ekpyrosis where the 4-dimensional action has been computed along with the trajectories of the cyclic universe in the pre-Ekpyrotic phase. Section 5 contains the curvature perturbations and their conversion to the entropy perturbations via adiabatic expansions that pave the way for computing the requisites of the potential amplitudes for the bounce or the phase transitions. Section 6 contains the instabilities, ghost condensations, and fluctuations on the effect of the pre-bounce phenomenology that nullifies at the Planks regime of the strings at the phase transitions from contractions to the expansions. This indicates the realistic phenomena of the catastrophic vacuum fluctuations which regard as the condensation phase for the Ghosts and new ghosts fields. On Section 7 the superfields and superpotentials of $N = 1$ supergravity has been rigorously

analysed. In Section 8, the SUSY Galileons and their ghost condensate connection to $N = 1$ Lagrangian are discussed while arriving finally at the conclusions.

3. Brane Worlds Cosmology & Ekpyrotic Universe

The authors in [33, 34] have beautifully described the cosmological models of Ekpyrotic theory based on M-Theory. A metric had been proposed by Randall and Sundrum^[see appendix A][35], that describes 2-Branes fixed at the boundaries of a 5-Dimensional universe, which can move along the 5th dimensions through the 11-Dimensional bulk. This is the idea of Ekpyrosis deeply rooted in superstring/M theory (Heterotic). The rest 11 – 1 temporal – 4 spatial = 6 spatial dimensions being compactified on a tiny scale that is irrelevant on the present cosmological scales. Imagine a universe, existing eternally but goes through a cyclic pattern that begins with an initial state characterized by the boundary Dirichlet(5)-Branes in a cold, flat, empty state. The two branes with a scalar field called the *radion flux* between them come close to each other and collide in such a non-singular way, that, it gives birth to a universe via a "new Ekpyrosis" called the 'big bang'. The energy from the collision creates the hot matter-fluid plasma in the branes, which then moves apart from one another and successively cools down. The *radion flux* which is a function of the distance between the two boundary 5-branes evolves through a period of slow acceleration, followed by deceleration and contraction. And then, again collides with a non-singular bounce [1, 2, 3] causing the reheating of the universe. Two important problems solved by Ekpyrosis is stated as,

- The homogeneity, that is the universe is the same in all directions, implies the CMBR is the same everywhere, that is, the universe begins with the same initial conditions at all points.
- The flatness problem could be solved by making the initial state of the branes a vacuum. As being in a state of vacuum, the branes are flat and empty, so, no mysterious fine-tuning is required to make the universe flat. The assumption asserts that the branes, that start in the vacuum force to be flat.

However, while viewing from a quantum scale, everything is not so exact, as it seems to be. There are brane fluctuations or *brane ripples* that arise because of the movements of the brane in the bulk. Not, every point in the brane, during 'bounce' collides at the same instant, some may collide after the other, while most will collide at an average time. This shows us, that rather than the whole universe be consistent with a uniform temperature, some regions are cooler (as the branes of that part collided earlier) than the others which are hotter (as the branes collided later times than the former). The seeds of creation lie in this phenomenology to create the large scale structures like the galaxies of the universe.

As discussed in section 2, the distasteful feature of a universe formation is the singularities which are space-time points where gravitational curvature and temperature blow up to infinity. One such example is the big bang singularity. However, in this model, the singularity is very milder than that of the classical general relativity, because before the branes collide, all ghost perturbations cut-off at the planks scale with a non-singular bounce, that bounces off and returns to the initial positions. The radiation and matter density of the branes are always finite and there are no singular points where, matter, space, and time sprung off by a magical fiat. The 'singular' behaviors are however observed in the big crunch when the two branes collide thereby disappearing the extra-dimensions between them which later re-emerges when the boundary branes move apart from each other. Of course, it's difficult to believe, that contrary to the normal supposition, space and time are always existent as in the Ekpyrotic models, but, more advanced theories and experiments will conclude the results with an exact precession.

The Ekpyrotic phenomenology also gives a block to the missing puzzle of "where all the matters of the universe come from?" The answer is, however, relatively simple, that is, during the brane to brane collision, a huge amount of thermal energy got created from the kinetic energy of the branes, which in turn condenses to matters via $E = mc^2$.

This cyclic model proposes the facts that,

- The universe is an eternally existent being that runs through repeated cycles of brane collisions.
- The big bang is not the origin of time.

As an alternative to inflations, this model solves various puzzles of the cosmological phenomena that may be distinguished by observational tests where the gravitational waves are not scale invariant, unlike inflation.

4. Scalar Potential of Ekpyrosis

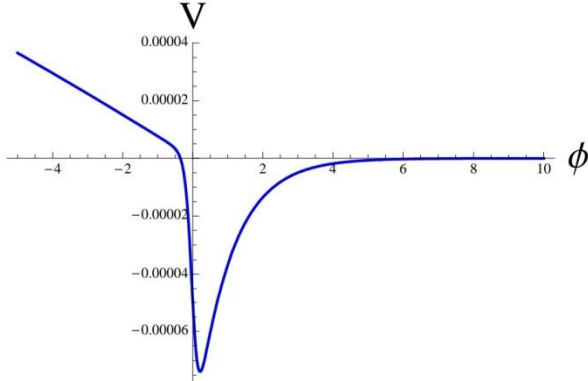


Figure 1: Generic diagram of the Ekpyrotic Scalar Potential. Figure from [2]

Generally, one effective scalar field is considered to describe a potential, but, here, instead of one, there are good reasons to consider 2 effective scalar fields [36]. Firstly, in the cyclic models of the M-Theory and embedding Ekpyrotic scenario, the radion fields, that determines the distance between the branes, and secondly, the bulk dimensions, where the volume modulus is a 6-dimensional manifold [37]. Shape moduli of the internal space can also be considered as a scalar field, yielding a 3rd potentials, but, it's potent to stick with the 2 universal scalar fields. In the case of the 2 scalar fields, the moduli of the scale-invariant curvature perturbations could be considered better than with only one. If we consider the 2 fields as $V(\phi_0)$ and $V(\phi_1)$, then the 4-dimensional effective action could be given by,

$$S = \int \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi_0)^2 - \frac{1}{2}(\partial\phi_1)^2 - V(\phi_0, \phi_1) \right] \quad (1)$$

Which is the Hořava-Witten theory of the low energy limit, where ϕ_0 is the radion and ϕ_1 is the internal volume modulus [38]. It has been assumed that during the Ekpyrotic phase, both the fields have a negative downward 'roll down' potentials through a sloppy gradient, for example,

$$V(\phi_0, \phi_1) = -V_0 e^{-c_0 \phi_0} - V_1 e^{-c_1 \phi_1} \quad (2)$$

Then, naturally, the dynamics can be discussed with the effect of the pointing transverse variable σ and pointing perpendicular variable ρ to the field velocity [39, 40]. They can be expressed as additive constants given by,

$$\sigma \equiv \frac{\dot{\phi}_0 \phi_0 + \dot{\phi}_1 \phi_1}{\dot{\sigma}}, \quad \rho \equiv \frac{\dot{\phi}_0 \phi_1 + \dot{\phi}_1 \phi_0}{\dot{\sigma}} \quad (3)$$

With $\dot{\sigma} \equiv \sqrt{\dot{\phi}_0^2 + \dot{\phi}_1^2}$. And the angle θ as the field space trajectory can be expressed as [41],

$$\cos \theta = \frac{\dot{\phi}_0}{\dot{\sigma}}, \quad \sin \theta = \frac{\dot{\phi}_1}{\dot{\sigma}} \quad (4)$$

Now, the potential can be re-expressed in terms of the new variables as,

$$V_{ekp} = -V_0 e^{\sqrt{-2\epsilon}\sigma} \left[1 + \epsilon \rho^2 + \frac{2\sqrt{2}(c_1^2 - c_2^2)/|c_1 c_2|}{3!} \epsilon^3 \rho^3 + \frac{4(c_1^6 - c_2^6)/c_1^2 c_2^2 (c_1^2 - c_2^2)}{4!} \epsilon^2 \rho^4 + \dots \right] \quad (5)$$

Where c is an arbitrary constant taken for the derivation, hence, the Ekpyrotic scaling solutions becomes,

$$a(t) = (-t)^{1/\epsilon}, \quad \sigma = -\sqrt{\frac{2}{\epsilon}} \ln(-\sqrt{\epsilon V_0 t}) \quad (6)$$

With the angular parameter θ being constant. Hence, the single field scale has shown a contrasting picture to the double field scale where the latter is unstable to small perturbations [42]. This shows the localization of the field potential at the ridge of the bounce in the Ekpyrotic phase with extreme precession having a value of $e^{-60 \pm 1}$ in Plank units at the onset of the Ekpyrosis. However, the initial condition still holds a problem that can be addressed in two ways as such,

- During the pre-Ekpyrotic phase, the field trajectory is curved upwards and localized.
- In the case of the cyclic universe, the trajectories that are close to the ridge, vastly amplified in successive cycles due to the domination of matter, dark energy, and radiation.

Now, at the same instant, the field trajectories that are not sufficiently close to the ridge undergo chaotic behaviors where they simply collapse to big crunch (and end up forming black holes [43, 44, 45]) and stop cycling and growing. In this way, the universe ends up with vast habitable regions with a small one interspersed with irregularities of collapse.

5. Curvature & Entropy Perturbations

The Newtonian potential \dot{a}'/a^3 has a relation with the growing mode of the field strength free parameter Φ that projects out of the curvature perturbations ξ_0 via a relation of direct approximation as $\Phi \sim \dot{a}'/a^3$ which asserts the curvature coefficients as [1],

$$\xi_0 = \frac{2}{3a^2(1+\kappa)} \left(\frac{\Phi}{a^3} \right) \quad (7)$$

Where κ represents the coefficients of the 'equation of the state' and is approximately equal to 1 at the time of non-singular 'bounce'. A crucial thing to say is that the physics of the bounce of the effective field theory lies within the domain of the 4-dimensions of gravity and matter from the contraction to the expansion. The mechanism of the perturbations of the 2-scalar fields [46, 47, 48] rolling down the steep exponential potential acquires entropy perturbations on scale invariance on large scales. If these entropy perturbations can be successfully endowed over the curvature perturbations, then this will result in a scale-invariant spectrum long before the bounce. Since this has been related to the adiabatic phase, much of the kinetic energy has been converted to thermal energy where the field potentials take a sharp trajectory with a scale invariant jump in ξ_0 . Therefore, the curvature perturbations parameter ξ_0 becomes unscathed whenever the physics of the bounce comes into play.

Sourced on the time derivative of ξ_0 , the entropy perturbations can be given by [1],

$$\dot{\xi}_0 \approx -2H \frac{\dot{\sigma}}{\sigma} \delta \rho \quad (8)$$

The curvature of the field trajectory makes the ξ_0 to take a sharp turn depending on the scales. Therefore, a conversion of the entropy perturbations takes place into the adiabatic mode as denoted by Figure 1. As expressed in section 4, if the gradient of the potential is steep, then for the scaling regime of the two field scenario, in a very short Hubble time, could be expressed by rewriting the equation (8) as,

$$\dot{\xi}_0 \approx -\frac{2H}{\sigma} \arctan\left(\frac{q}{p}\right) \delta(t - t_i) \delta \rho \quad (9)$$

Where $q \ll 1$ and the transition time of the bounce is t_i . Thus, the transition begins at $t = t_i$ and ends at $t = t_i + \mu$, with $H\mu \ll 1$. Choosing $\theta_i = \arctan\left(\frac{q}{p}\right)$ and $\theta_f = 0$, the equation of the amplitude $(\delta\rho)_i$ and the time derivative of the amplitude $(\dot{\delta\rho})_i$ at the phase of the transition can be expressed as [1],

$$\delta\rho(t) = (\delta\rho)_i \cos\left(\frac{\sqrt{3}(\theta_f - \theta_i)(t - t_i)}{\mu}\right) + \frac{\mu}{\sqrt{3}(\theta_f - \theta_i)} (\dot{\delta\rho})_i \sin\left(\frac{\sqrt{3}(\theta_f - \theta_i)(t - t_i)}{\mu}\right) \quad (10)$$

Where, $(\dot{\delta\rho})_i \sim H(\delta\rho)$, and therefore, $(\dot{\delta\rho})_i \mu \ll (\delta\rho)_i$, with as a case of interest $|\theta_f - \theta_i| \approx \arctan\left(\frac{q}{p}\right)$ proves that $\delta\rho$ changes by a factor of unity during the transition. The final equation is given by,

$$(\delta\rho)_f \approx (\delta\rho)_i \cos\left(\sqrt{3}(\theta_f - \theta_i)\right) \quad (11)$$

6. Ghosts, Instabilities & Catastrophic Vacuum Fluctuations

Two scalar potentials are needed in section 4 for the development of a successful Ekpyrotic cosmology, however, for the sake of simplification, let us consider only one potential to develop a sustainable equation of the ghost condensate. Let the potential be $V(\phi)$ and as indicated in section 2, the damping of the higher derivative term associated with the ghost phase be $-(\square\phi)^2$. Now, it's safe to proceed with the equation [2, 49, 50],

$$G_{cdns} = \sqrt{g} \left[M^4 P(X) - \frac{1}{2} \left(\frac{\square\phi}{M'} \right)^2 - V(\phi) \right] \quad (12)$$

Where the first two terms are the ghost condensates and the last term is the single scalar potential. The condensate varies on the scalar fields parameter ϕ , so, for large values of ϕ , the theory has been reduced to the ghost condensate model. This condensate attains minimum when $X = 1/2$, where the fraction at the minimum potential of the condensate having the values of the first term of equation (12) looks like,

$$P(X) = \frac{1}{2} \left(X - \frac{1}{2} \right)^3, \quad X \neq 0 \quad (13)$$

The term $-\frac{1}{2} \left(\frac{\square\phi}{M'} \right)^2$ has been added with the LHS of equation (13) yielding equation (12), to dampen the fluctuation of the large negative scalar field ϕ .

The Einstein equation corresponds, the ghost equation in the form [2],

$$\dot{H} = -\frac{1}{2}(\varepsilon + p) = -M^4 P_{,X} X = -M^4 X(X - 1/2) \quad (14)$$

Where the energy density is ε and the pressure is p . NEC requires that $\varepsilon + p \geq 0$ and $\dot{H} \leq 0$ while the NEC gets violated at $\varepsilon + p < 0$, this implicitly shows us, that bounce can only be possible when $P_{,X} < 0$ that is, the X must be smaller than $1/2$. Ignoring the gravitational effects $H = 0$, expansion rate $\dot{a} = 1$, as we are only interested in the rapidly growing instability, the perturbed instability I_0 for the field potential ϕ could be addressed with a small value of $P(X)$ as [2],

$$G_{cdns} = \frac{M^4}{m_g^4} \left[\frac{1}{2} \dot{I}_0^2 - \frac{1}{2} P_{,X} (\nabla I_0)^2 - \frac{1}{2m_g^2} (\square I_0)^2 \right] \quad (15)$$

Now, to analyze the instability at a small frequency, the dispersion relation becomes [2, 49, 50],

$$\omega^2 = P_{,X} (k)^2 + \frac{k^4}{m_g^2} \quad (16)$$

This expression shows the perfect assumptions of the new Ekpyrotic scenario, that, in a limited range of momentum k , if the parameter m_g^2 is small, then, the higher derivative would be large which could be the canceling of the instability by the addition of the term $-\frac{1}{2m_g^2} (\square I_0)^2$. The

analytical investigation shows, that, because of the existence of the ghosts, the theory suffers from a large vacuum instability, but this has a classical significance as this relates to the violation of the NEC: hence, the higher derivatives which have been developed cancels out the ghost condensate mass, leading to a less perturbed theory of Ekpyrotic cosmology.

If we consider a collapsing universe of matter density ρ_M and an ultra-relativistic ghost gas of $-\rho_g < 0$, then, at the scale of the expansion parameter \dot{a} , we can say that the absolute density of the ghost grows farther than the matter density as $\rho_M - \rho_g > 0$. Thus, the ghosts would be useful, in such a case that, the unitarity problem has been replaced by the vacuum instability in such a way as the negative energy of ghosts cancels out the positive energy of matter as seen in the equation $\rho_M - \rho_g > 0$ with a little abundance of matter left after the big bang to create the universe along with the large scale structures that too, without violating NEC [51, 52, 53, 54, 55, 56].

As in [57], at a large value of the scalar potential, the universe enters an Ekpyrotic contraction phase, having the potential limit,

$$V(\phi) = -V_0 \left(\frac{1}{2} \left[1 + \tanh\left(\lambda(\phi - \phi_{ekpyrotic \text{ -end}})\right) \right] \right) e^{-c(\phi)\phi} \quad (17)$$

Where, c takes the value of $> \sqrt{6}$ satisfying the equation of the state as $\kappa = 1$. At this time the kinetic term gets approximately canonical, and the universe contracts slowly where the anisotropies are suppressed. The universe again speeds up to a normal kinetic phase as $\phi = \phi_{ekpyrotic \text{ -end}}$ where the potential rises back from the bottom to the zero. The energy density of the scalar field that lies in the equation of state $\kappa \approx \text{pressure/density}$ is dominated and the sign-switching of the kinetic terms occur, that in essence gives the ghost condensate and the Galileon term with a violation of the NEC as follows,

$$\dot{\phi}^2 < \frac{1}{\tilde{\tau}} \quad (18)$$

Such that, the universe bounces then at a small value of $V(\phi)$. After the bounce the universe starts to expand with the canonical kinetic term, again comes into play. The kinetic term of the bounce $k(\phi)$ is unity everywhere except when its near to the potential $V(\phi) = 0$ and is expressed by,

$$k(\phi) = 1 - (2(1 + 2k(\phi)^2)^{-2}) \quad (19)$$

The parameter k denotes the switch of the sign of the kinetic term and is essentially the width of the phase that being subject to switching, where the Galileon term $g(\phi)$ and the boundary of the kinetic term $\tau(\phi)$ has a relation with the $k(\phi)$ such that, they can be expressed as,

$$g(\phi) = \tilde{g}(1 + 2k(\phi)^2)^{-2}, \quad \tau(\phi) = \tilde{\tau}(1 + 2k(\phi)^2)^{-2} \quad (20)$$

It is crucial the equation 20, is always non-zero, because when equation 19 passes through zero, it otherwise would develop a singularity.

7. $N = 1$ Supergravity Extensions

To express the cosmological model with the canonical supergravity formalism [57, 58, 59], we have ignored the fermionic contributions while applying to the ghost contribution model. The bosonic components can be expressed by the Lagrange function as,

$$\begin{aligned} \mathcal{L}^{(BOSONS)} = & -\frac{1}{8} \int d^2\theta 2\mathcal{E}(\tilde{\mathcal{D}}^2 - 8R) \left[-3e^{-\frac{K(\Phi, \Phi^\dagger)}{3}} \right. \\ & + \left(\mathcal{D}^\beta \Phi \mathcal{D}_\beta \Phi \tilde{\mathcal{D}}_\beta \Phi^\dagger \tilde{\mathcal{D}}^\beta \Phi^\dagger T(\Phi, \Phi^\dagger, \partial_m \Phi, \partial_n \Phi^\dagger, \dots) \right) \\ & + \left. \left(\mathcal{D}^\beta \Phi \mathcal{D}_\beta \Phi \tilde{\mathcal{D}}_\beta \Phi^\dagger \tilde{\mathcal{D}}^\beta \Phi^\dagger \tilde{\mathcal{G}}(\Phi, \Phi^\dagger) \right) \right] + H.c. a \\ & + \int d\theta 2\mathcal{E} W(\Phi) + H.c \end{aligned} \quad (21)$$

Where Φ is the chiral superfield with the following components, $\Phi \equiv A$, $\mathcal{D}^\beta \Phi \equiv -\frac{1}{4}F$ with tilde denotes the lower components having A, F as scalar supersymmetric fields, $W(\Phi)$ is the superpotential of a holomorphic functions of (Φ) , K in $K(\Phi, \Phi^\dagger)$ being the Kähler potential, having the Hermitian function that determines the two derivative of the kinetic fields A and Φ . $T(\Phi, \Phi^\dagger, \partial_m \Phi, \partial_n \Phi^\dagger, \dots)$ is the tensor superfield of the chiral and anti-chiral matters and their covariant derivatives, having all the indices contracted.

The corresponding bounce fields, as expressed by the two scalar potentials as Φ_0 and Φ_1 with the equation of state being $\kappa \ll 1$, the Lagrangian becomes[57],

$$\begin{aligned} \frac{1}{e} \mathcal{L}^{(bounce)} &= -\frac{1}{2} \mathcal{R} - \frac{1}{2} K_{,AA^*} [((\partial\phi_0)^2 + (\partial\phi_1)^2)] \\ &+ [(\partial\phi_0)^4 + (\partial\phi_1)^4 - 2(\partial\phi_0)^2(\partial\phi_1)^2 \\ &+ (\partial\phi_0 \cdot \partial\phi_1)^2] \\ &* \left[T + \frac{4}{3} G(K_{,A} + K_{,A^*}) - \frac{32}{3} \check{G}^2 (\partial\phi_1)^2 \right] \\ &+ 4\sqrt{2} \check{G} [(\partial\phi_0)^2 \square\phi - (\partial\phi_1)^2 \square\phi + 2\partial\phi_0 \cdot \partial\phi_1 \square\phi_1] \end{aligned} \quad (22)$$

Where $K_{,AA^*} = K(\phi)$ and $G(\phi_0, \phi_1) = -\frac{1}{8\sqrt{2}} g(\phi)$.

Similarly, the Lagrangian of the Ekpyrotic phase can be given by[57],

$$\frac{1}{e} \mathcal{L}^{(Ekpyrotic)} = -\frac{1}{2} \mathcal{R} - K_{,AA^*} (\partial A \cdot \partial A^*) - e^K (K^{AA^*} |D_A W|^2 - 3|W|^2) \quad (23)$$

Omitting all the terms of the fermions, to rescale the Lagrangian from the canonical two derivative Weyl form of supergravity from the Einstein frame, this contributes as $\mathcal{L}^{(WEYL)} \rightarrow \mathcal{L}^{(X)} + \mathcal{L}^{(X^2)} + \mathcal{L}^{(GALILEON)}$ corresponds to [57],

$$\begin{aligned} \frac{1}{e} \mathcal{L}^{(WEYL)} &= -\frac{1}{2} R - \frac{1}{12} K^{AA^*} K_{,AA^*} + 3e^{\frac{K}{3}} \left(\frac{\partial^2 e^{-\frac{K}{3}}}{\partial A \partial A^*} \right) (\partial A \partial A^*) - 3e^{\frac{2K}{3}} \left(\frac{\partial^2 e^{-\frac{K}{3}}}{\partial A \partial A^*} \right) |F|^2 \\ &+ \frac{1}{3} b^m b_m - \frac{i}{3} b^m (K_{,A} A_{,m} - K_{,A^*} A^*_{,m}) - \frac{1}{3} e^{\frac{K}{3}} M^* F^* K_{,A^*} \\ &- \frac{1}{3} e^{\frac{K}{3}} |M|^2 - e^{\frac{2K}{3}} W M^* - e^{\frac{2K}{3}} W^* M + e^{\frac{2K}{3}} W_{,A} F + e^{\frac{2K}{3}} W^*_{,A^*} F^* \\ &+ 8 \left((\partial A)^2 (\partial A^*)^2 - 2e^{\frac{K}{3}} |F|^2 (\partial A \partial A^*) + e^{\frac{K}{3}} |F|^4 \right) (T|W + T|W^*) + 8(\partial A)^2 A^* G + 8(\partial A^*)^2 A G^* \\ &+ \frac{8}{3} (\partial A)^2 K^m A^*_{,m} G + \frac{8}{3} (\partial A^*)^2 K^m A_{,m} G^* + i \frac{16}{3} (\partial A)^2 b^m A^*_{,m} G - i \frac{16}{3} (\partial A^*)^2 b^m A_{,m} G^* \\ &+ i \frac{32}{3} e^{\frac{K}{3}} |F|^2 b^m A_{,m} G - i \frac{32}{3} e^{\frac{K}{3}} |F|^2 b^m A^*_{,m} G^* + \frac{16}{3} e^{\frac{K}{3}} M^* F^* (\partial A \partial A^*) G^* \\ &- \frac{8}{3} e^{\frac{2K}{3}} M F |F|^2 G - \frac{8}{3} e^{\frac{2K}{3}} M^* F^* |F|^2 G^* + 16e^{\frac{K}{3}} F^* F^m A_{,m} G + 16e^{\frac{K}{3}} F F^*{}^m A^*_{,m} G^* \\ &- 2e^{\frac{K}{3}} (\partial A^*)^2 F^* (\mathcal{D}^2 \check{G})^W \Big| - 2e^{\frac{K}{3}} (\partial A)^2 F (\mathcal{D}^2 \check{G})^{\dagger W} \Big| + 2e^{\frac{2K}{3}} |F|^2 F^* (\check{\mathcal{D}}^2 \check{G})^{\dagger W} \Big| \\ &- i 4e^{\frac{K}{3}} A_{,m} \sigma_{a\dot{a}}^m (\mathcal{D}^a \check{\mathcal{D}}^{\dot{a}} G)^W \Big| + i 4e^{\frac{K}{3}} A^*_{,m} \sigma_{a\dot{a}}^m (\mathcal{D}^a \check{\mathcal{D}}^{\dot{a}} G)^{\dagger W} \Big| \end{aligned} \quad (24)$$

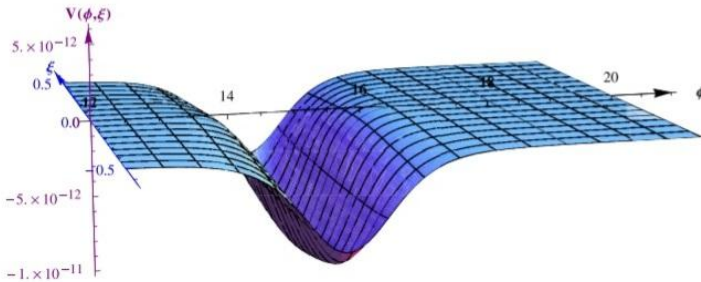


Figure 2. The Ekpyrotic superpotential has been defined in terms of the potentials $V(\phi, \xi)$ which is presented in the paper as $V(\phi_0, \phi_1)$ where $V(\phi_1) = 0$ for $b = \sqrt{3}$, $d = 1$ that in essence shows the decreasing potential $\phi_{ekpyrotic} -end$ with a negative gradient that falls steeply with a saddle-like curvature and bounces back with a non-locally flat curvature satisfying the FLRW^[see Appendix B] metric with a small local positive curvature. Source [57].

The supersymmetric ghost condensate theory with a superpotential involving two scalar fields, when coupled to $N = 1$ supergravity yields locally SUSY chiral actions which admits a ghost condensate in a de Sitter space-time. Expanding around this vacuum, the theory can be shown to be absent of any ghost without spatial gradient instabilities. To take into account the supergravity extensions [59] of the ghost condensate $P(X) = -X + X^2$ the following correlations could be defined as,

$$K(\phi, \phi^\dagger) = -\phi\phi^\dagger, \quad \tau = 1 \quad (25)$$

Taking the ϕ_1 as the potential of the kinetic term, and defining $A = \frac{1}{\sqrt{2}}(\phi_0 + i\phi_1)$, the Lagrangian becomes [59],

$$\frac{1}{e} \mathcal{L}_{T^4}^{SUGRA} = -\frac{1}{2} \mathcal{R} + \frac{1}{2} (\partial\phi_0)^2 + \frac{1}{4} (\partial\phi_0)^4 + \frac{1}{2} (\partial\phi_1)^2 + \frac{1}{4} (\partial\phi_1)^4 - \frac{1}{2} (\partial\phi_0)^2 (\partial\phi_1)^2 + (\partial\phi_0 \cdot \partial\phi_1)^2 + \dots \quad (26)$$

Where the remaining quadratic terms κ, ψ_m are the fermions which has been set to zero, for a vanishing gravitino as stated in equation (21), with the scaling factor (in the first slice) $\hat{a} \sim a(t) = e^{\pm \frac{t}{\sqrt{12}}}$, one must set $c = 1$ in the metric $ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$ with the \pm sign corresponds to the expanding and contracting phase respectively.

As seen in the formalism of [59, 60, 61], global $N = 1$ supersymmetry has been extended in the scalar ghost condensate by virtue, of the superspace derivatives that could be defined through a complex scalar potential A as seen in equations (22, 23, 24) through space-time coordinates x^m and the Grassmann spinor coordinates $\Psi^\alpha, \bar{\Psi}_\alpha$. The highest component of the complex scalar field $A(x)$, auxiliary field $F(x)$ and a spinor component $\zeta_\alpha(x)$ with the highest component being $\Psi, \bar{\Psi}, \bar{\Psi}$ being invariant under supersymmetry transformations could be helped to provide the supersymmetric Lagrangian of a four superspace differential $d^4\Psi \equiv d^2\Psi d^2\bar{\Psi}$ given in terms of the components of the Kähler potential given by,

$$\mathcal{L}_{\Phi, \Phi^\dagger} = \int d^4\Psi \Phi \Phi^\dagger = \Phi \Phi^\dagger|_{\Psi, \bar{\Psi}, \bar{\Psi}} = -\partial A \cdot \partial A^* + F^* F + \frac{i}{2} (\zeta_m \sigma^m \bar{\zeta} - \zeta \sigma^m \bar{\zeta}_m) \quad (27)$$

Now, defining the complex scalar field $A(x)$ with the two real components as shown from equation (1) onwards is given ϕ_0, ϕ_1 as [59],

$$A(x) = \frac{1}{\sqrt{2}} (\phi_0 + i\phi_1) \quad (28)$$

The Lagrangian becomes,

$$\mathcal{L}_{\Phi, \Phi^\dagger} = -\frac{1}{2} (\partial\phi_0)^2 - \frac{1}{2} (\partial\phi_1)^2 + F^* F + \frac{i}{2} (\zeta_m \sigma^m \bar{\zeta} - \zeta \sigma^m \bar{\zeta}_m) \quad (29)$$

With the SUSY extensions of $X = -\frac{1}{2} (\partial\phi)^2$ the globally supersymmetric field can be extended to X^2 with the spinor $\zeta_\alpha(x)$ defined as [59, 60, 61],

$$\begin{aligned} \mathcal{L}_{\mathcal{D}\Phi\mathcal{D}\Phi\mathcal{D}\Phi^\dagger\mathcal{D}\Phi^\dagger} &= \frac{1}{16} \mathcal{D}\Phi\mathcal{D}\Phi\mathcal{D}\bar{\Phi}\mathcal{D}\bar{\Phi}^\dagger|_{\Psi, \bar{\Psi}, \bar{\Psi}} \\ &= (\partial A)^2 (\partial A^*)^2 - 2F^* F \partial A \cdot \partial A^* + F^{*2} F^2 - \frac{i}{2} (\zeta \sigma^m \bar{\sigma}^l \sigma^n \bar{\zeta}_m) A_{,m} A_{,l}^* \\ &+ \frac{i}{2} (\zeta_{,n} \sigma^n \bar{\sigma}^m \sigma^l \bar{\zeta}) A_{,m} A_{,l}^* + i\zeta \sigma^m \bar{\sigma}^n A_{,m} A_{,n}^* - i\zeta^m \sigma^n \bar{\zeta}_m A_{,n}^* \\ &+ \frac{i}{2} \zeta \sigma^m \bar{\zeta} (A_{,m}^* \square A - A_{,m} \square A^*) + \frac{1}{2} (F \square A - \partial F \partial A) \bar{\zeta} \bar{\zeta} + \frac{1}{2} (F^* \square A^* - \partial F^* \partial A^*) \zeta \zeta \\ &+ \frac{1}{2} F A_{,m} (\bar{\zeta} \bar{\sigma}^m \sigma^n \bar{\zeta}_{,n} - \bar{\zeta}_{,n} \bar{\sigma}^m \sigma^n \bar{\zeta}) + \frac{1}{2} F^* A_{,m}^* (\zeta_{,n} \sigma^n \bar{\sigma}^m \sigma^l \bar{\zeta} - \zeta \sigma^n \bar{\sigma}^m \zeta_{,n}) \\ &+ \frac{3i}{2} F^* F (\zeta_m \sigma^m \bar{\zeta} - \zeta \sigma^m \bar{\zeta}_m) + \frac{i}{2} \zeta \sigma^m \bar{\zeta} (F F^*_{,m} - F^* F_{,m}) \end{aligned} \quad (30)$$

Now, the potentials ϕ_0, ϕ_1 could be expressed in terms of A and A^* as [59],

$$(\partial A)^2 (\partial A^*)^2 = \frac{1}{4} (\partial\phi_0)^4 + \frac{1}{4} (\partial\phi_1)^4 - \frac{1}{2} (\partial\phi_0)^2 (\partial\phi_1)^2 + (\partial\phi_0 \cdot \partial\phi_1) \quad (31)$$

8. Supersymmetric Galileons

Galileon theories have an implicit relation with the ghost condensate as they allow for stable violations of the NEC and are of considerable interest to study the high energy regime at the pre-Big bang epoch of the universe with $N = 1$ SUSY theories. The non trivial point is that, they admit non-negative stress-energy tensors for manifestations of a considerable non-stable vacua condition of NEC.

The general Lagrangians for NEC as described in [62, 63] have been shown where c_i are constant coefficients, restricting to time dependent fields allowing the equations,

$$\mathcal{L} = c_2 \mathcal{L}_2 + c_3 \mathcal{L}_3 + c_4 \mathcal{L}_4 + c_5 \mathcal{L}_5 \quad (32)$$

Where the relation between $\mathcal{L}^{GALILEONS}$ and $P(X)$ could be expressed as,

$$\mathcal{L} = \frac{1}{\phi^4} P(X), \quad P(X) = c_2 X - c_3 X^2 + \frac{6}{5} c_4 X^3 - \frac{6}{7} c_5 X^4 \quad (33)$$

That is, the conformal Galileon models are seen to be identical upto the scale parameter of $\frac{1}{\phi^4}$ – where the connection satisfies the two theories, on the basis of a time independent solutions in a ghost condensate scenario of the pre-Big bang epoch.

With the connection of $P(X, \phi)$ theories, the respective Lagrangians $\mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_4, \mathcal{L}_5$ could be presented via the equations [62, 63] as,

$$\begin{aligned}
 \mathcal{L}_2 &= -\frac{1}{2\phi^4}(\partial\phi)^2 \\
 \mathcal{L}_3 &= \frac{1}{2\phi^3}\square\phi(\partial\phi)^2 - \frac{3}{4\phi^4}(\partial\phi)^4 = -\frac{1}{4\phi^4}(\partial\phi)^4 - \frac{1}{6\phi^2}(\partial_\mu\partial_\nu\phi)^2 + \frac{1}{6\phi^2}(\square\phi)^2 \\
 \mathcal{L}_4 &= -\frac{1}{2\phi^2}(\partial\phi)^2(\square\phi)^2 + \frac{1}{2\phi^2}(\partial\phi)^2\phi^{\mu\nu}\phi_{,\mu\nu} + \frac{4}{5\phi^3}(\partial\phi)^4\square\phi - \frac{4}{5\phi^3}(\partial\phi)^3(\partial\phi)^2\phi_{,\mu\nu}\phi^{,\mu\nu} - \frac{3}{20\phi^4}(\partial\phi)^6 \\
 \mathcal{L}_5 &= (\partial\phi)^2\left[\frac{1}{2\phi}(\phi)^3 + \frac{1}{\phi}\phi^{\mu\nu}\phi_{,\nu\sigma}\phi_{,\mu}^{\sigma} - \frac{3}{2\phi}\phi\phi^{\mu\nu}\phi_{,\mu\nu} - \frac{3}{4\phi^2}\partial_\mu\partial^\mu(\partial\phi)^2(\partial\phi)^2 + \frac{3}{\phi^2}\phi\phi^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + \frac{6}{7\phi^3}(\partial\phi)^2\phi^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - \frac{6}{7\phi^3}(\partial\phi)^4\phi - \frac{3}{56\phi^4}(\partial\phi)^8\right] \\
 &= (\partial\phi)^2\left[\frac{1}{2\phi}(\square\phi)^3 + \frac{1}{\phi}\phi^{\mu\nu}\phi_{,\nu\sigma}\phi_{,\mu}^{\sigma} - \frac{3}{2\phi}\square\phi\phi^{\mu\nu}\phi_{,\mu\nu} - \frac{3}{4\phi^2}(\partial\phi)^2(\square\phi)^2 + \frac{3}{4\phi^2}(\partial\phi)^2\phi^{\mu\nu}\phi_{,\mu\nu} + \frac{9}{14\phi^3}(\partial\phi)^4\square\phi - \frac{9}{14\phi^3}(\partial\phi)^2\phi^{\mu\nu}\phi_{,\mu\nu} - \frac{3}{56\phi^4}(\partial\phi)^8\right]
 \end{aligned}
 \tag{34}$$

9. Discussion

The Cyclic universe model of the new Ekpyrotic scenario has been developed as an alternative to inflation to provide a view to the physicists about the state of the universe before the big bang. However, the bounce or the phase transition from the contraction to the expansion has been developed with the two scalar potential fields, and the entropy, curvature perturbations which involved a catastrophic vacuum instability as the ghost. This paper reflects not only the positive side of the ghost condensate period, but also it shows the way to *tame the ghosts* and made the physics of the universe in a positive matter/energy dominated region free from all the ghost like instabilities. And, all this at the cost of the superpotentials that entails as a Lagrangian through the forms of bouncing, Ekpyrosis, Weyl formalism to show the effectiveness of the bosonic gravity and its relation to the ghost potentials of two independent scalar fields.

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Appendix A

The Randall-Sundrum action is given by the metric [33],

$$S_{(R-S\ MDL)} = \int dy d^4x \sqrt{-g} \left(\frac{M_5^3}{2} R - \Lambda \right) + \sum_{i=1}^2 \int d^4x \sqrt{-h^{(i)}} (\Lambda_i + L_{\text{matter}}^{(i)})$$

The extra spatial dimension is denoted by y and the space-time coordinates has been referred to as x^u with a 5D metric g_{AB} . The metric of the 2-branes are given by $h^{(i)}_{\mu\nu} = g_{\mu\nu}(x^\mu, y_i)$ where $i = 1$ for the visible brane and $i = 2$ for the hidden brane. The additional terms are,

- M_5 is the Plank mass in 5D.
- Λ is the cosmological constant in the bulk.
- Λ_1 is the cosmological constant on visible brane while Λ_2 is the cosmological constant on the hidden brane.
- R is the scalar curvature in 5D.
- $L_{\text{matter}}^{(i)}$ is the Lagrangian density of the matter fields on the visible brane where it is the standard model, but on the hidden brane, it could be different.

The dimension y has a range value of $0 \leq y \leq \pi r_c$ where the two boundary branes are far apart with the r_c as a constant which indicates the visible brane at $y = \pi r_c$, while the invisible brane at $y = 0$.

This action helps to solve the hierarchy problem of the standard model with two branes *Plank* and *TeV* separated apart in the high dimensional bulk space-time.

Appendix B

FLRW or Friedmann-Lemaitre-Robertson-Walker metric is a solution of the Einstein-field-equations that describes a homogeneous, isotropic, path-connected expanding(or contracting) universe. The metric for the curvature normalized coordinates is gen as a proportion to the radial distance r as,

$$a(t)^2 d\Sigma^2 = dr^2 + S_k(r)^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

Where,

$$S_k(r) = \begin{cases} \sqrt{k}^{-1}, & k > 0 - \text{elliptic curvature} \\ r, & k = 0 - \text{Euclidean curvature} \\ \sqrt{|k|}^{-1}, & k < 0 - \text{hyperbolic curvature} \end{cases}$$

And, $a(t)$ denotes the scales of the commoving objects in space-time which either contracts or expands as the exact solutions of the general theory of relativity.
