New equations for energy dissipation down a stepped spillway

ABSTRACT

Stepped spillways use their stepping nature to dissipate energy in floodwater. Many researchers investigated the hydraulic and geometric relationships of the stepped spillway with slopes above 26.6^{o} degrees, which resulted in energy dissipation. Few, however, studied stepped spillways with slopes below 26.6^{o} degrees, which resulted in energy dissipation with fewer proposing models that estimated its energy losses. The authors reviewed researchers' publications on horizontal stepped spillways with slopes between $3.4^{o} \leq \theta \leq 26.6^{o}$ conducted in skimming flows in large-size facilities with phase-detection intrusive probes. They obtained data sets from them, which they reanalyzed to develop two new No-energy dissipation models that govern skimming flows over a wide range of operating conditions. The new models generated data compared well with the measured data with high coefficients of correlation between 0.95 and 0.99. All the data agree independent of channel slopes and sensor size. The models are simple, easy to use, and render more accurate results than the existing model.

Keywords: Aerated flow, energy dissipation, chute slope, dam height, stepped spillway, skimming flow.

1. INTRODUCTION

Major damage may occur if the energy of floodwater, especially its kinetic energy, is not dissipated safely. One type of flood release facility is the stepped spillway. Their steps offer significant resistance to flow, which result in energy dissipation. This loss in energy leads to the design of smaller and more economical dissipation structures downstream of the chute. As the discharge down the chute increases, a critical value would be reached, beyond which air would enter into the flow. This phenomenon is known as a two-phase flow or Air-Water flow.

Researchers like Horner (1969); Peyras et al. (1992); Chanson (1995, 2001); Ohtsu & Yasuda (1997); Chamani & Rajaratnam (1999); Boes (2000); Matos (2001); Toombes (2002) in the last decades, experimentally investigated air-water flows on stepped spillways with a dam slope above 26.6^{O} degrees. They provided vast information and guidelines for its design.

Few, however, studied stepped spillways with slopes below 26.6^{o} degrees, which resulted in energy dissipation with fewer proposing models that estimated its energy losses. This result in limited information and guidelines for designers of stepped spillways with slopes below 26.6o. There is also a lack of design rules and publications for stepped spillways; Hence, the need for an additional study on stepped spillways. Thus, the authors conducted a study that resulted in better equations and provided design engineers with the knowledge of the impact of a stepped spillway on energy dissipation performance.

Researchers identified three kinds of flows that occur over a stepped spillway. Such flows include:
a) nappe flow regime, b) transition flow regime, and c) skimming flow regime.

In the nappe flow regime, a sequence of water flows down from one step to the next lower step with the formation of a hydraulic jump at each step. This flow is like a sequence of separate drop structures linked together (Chamani and Rajaratnam, 1994; Chanson, 1993). Energy losses accompany the hydraulic jump. The losses occur due to
a) The disintegration of the spout in the air, and
b) The blending of flow on the steps, with or without the development of hydraulic jump on the step (Chanson, 1994; Rajaratnam, 1990) and these energy losses could be calculated using equations [1.1] or [1.2].

Nappe flow with an established hydraulic jump (Figure 1) usually occurs from small discharges with shallow flow depths. The flow cascades over steps, with the formation of supercritical flow at the edges of the steps, and returns to subcritical flow downstream of the jump.

In the nappe flow regime, a sequence of drops from one step to the next lower step occurs with the formation of a hydraulic jump on each step. This flow type is like a sequence of separate drop structures (Chamani and Rajaratnam 1994; Chanson 1993).

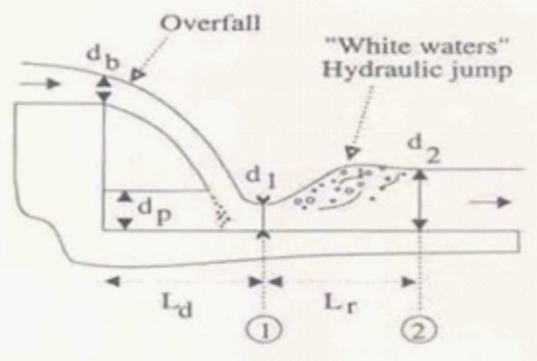

Figure 1: Nappe flow regime (Flow at a drop structure)

The water flows over one step of the spillway to the next lower step with energy loss occurring from: a) the disintegration of the jet in the air and b) the mixing of the flow on the steps, with or without the development of hydraulic jump, on the step (Chanson, 1994; Rajaratnam, 1990). The energy losses could be computed using equations [1.1] or [1.2].

$$\frac{\Delta H}{H_o} = 1 - \frac{\frac{d_1}{d_c} + \frac{1}{2}\left(\frac{d_c}{d_1}\right)^2}{H_{max} + \frac{3}{2}} \qquad [1.1]$$

Where d_1 is the water depth at impact, d_c is the critical water depth, and H_{max} is the dam height, ΔH is the energy loss, H_o is the maximum available energy, h is the height of the spillway step. Chanson (1994) later expressed this equation in terms of the spillway step height, the critical flow depth, and the dam height as:

$$\frac{\Delta H}{H_o} = 1 - \left[\frac{0.54\left(\frac{d_c}{h}\right)^{0.275} + \frac{3.43}{2}\left(\frac{d_c}{h}\right)^{-0.55}}{\frac{3}{2} + \frac{H_{dam}}{d_c}}\right] \qquad [1.2]$$

Nappe flow with completely established hydraulic jump (Figure 1), usually arises from small discharges with shallow flow depths and flow over the step with formation of supercritical at the edge of the step and returns to subcritical flow downstream of the jump.

In the skimming flow regime, the flow occurs with the submergence of the steps with the development of a fully aerated uniform flow downstream of a long chute. Along the upstream steps, a non-aerated flow region exists in which a turbulent boundary layer develops. Air entrainment in the flow begins where the boundary layer intersects the free surface, referred to as the point of inception. Downstream of the point of inception, the flow continues to aerate and varies gradually in depth (Figure 2). The flow eventually becomes a fully aerated uniform flow in which the water depth, velocity, and air concentration become constant (Bindo et. al 1993) (Figure 3). If uniform flow conditions are reached downstream of the spillway, this energy loss could be computed as follows

In skimming flow, most energy dissipation occurs to keep stable depression vortices.

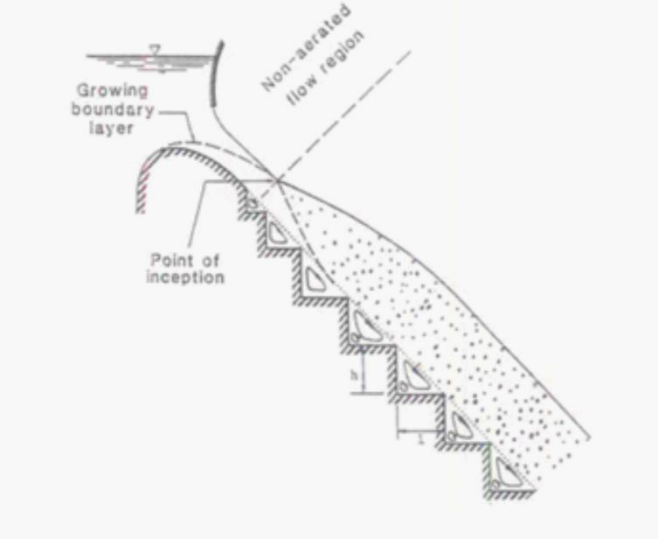

Fig. 2. Skimming flow regime - Sorensen (1985)

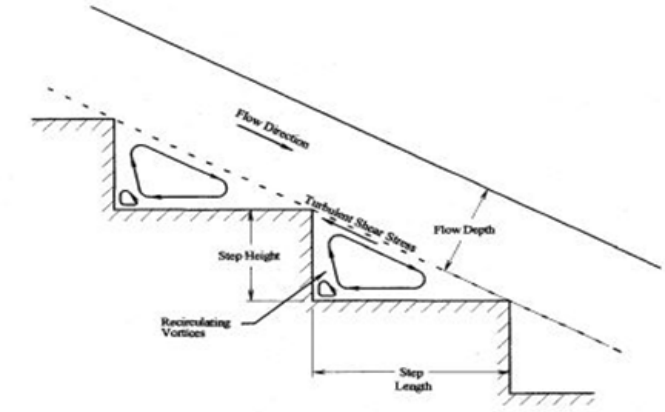

Fig. 3. Skimming flow regime with uniform flow conditions

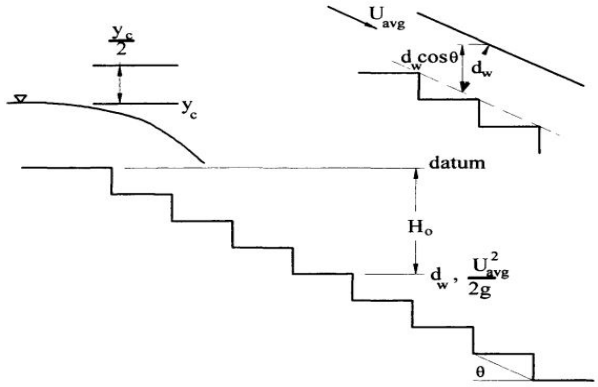

Fig. 4. Arrangement of the spillway with the definition of the variables

In skimming flow, most of the energy is dissipated in the maintenance of stable depression vortices. If uniform flow conditions are reached at the downstream end of the spillway, this energy loss could be computed as follows

$$\frac{\Delta H}{H_{max}} = \frac{d_w \cos\theta + \frac{U_{avg}^2}{2g}}{Nh + \frac{3}{2}d_c} \qquad [1.3]$$

Where d_w is the clear water depth, U_{avg} is the average velocity, the total head loss may be rewritten in terms of the friction factor, f, the spillway slope, θ, in degree, the critical depth, d_c, and the dam height, H_{dam}:

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\left(\frac{f}{8sin\theta}\right)^{1/3} cos\theta + \frac{E}{2}\left(\frac{f}{8sin\theta}\right)^{-2/3}}{\frac{H_{dam}}{d_c} + \frac{3}{2}} \qquad [1.4]$$

Eq [1.4] was computed for spillway slope with θ = 52 (degrees) and friction factor, f = 0.03 and f = 1.30, that represent average flow resistance on smooth spillways and stepped spillways, respectively. where E is the kinetic energy correction coefficient, ϑ is the dam slope in degrees.

2. MATERIALS AND METHODS

The authors carefully selected from the literature 11 No publications on horizontal stepped spillways with minimum step heights of 3 cm, undistorted Froude values and large dimensionless discharges corresponding to Reynolds numbers between 1×105 and 1×106 (Table 1)

These researchers used experimental facilities on large scale to minimize scale effects that affect the air-water flow processes in high-velocity free-surface flows [15]. Scale effect is defined as slight misrepresentations, which occurs when secondary forces like viscous forces and surface tension forces in turbulent flow are ignored. They are usually overlooked in many open-channel flows. However, if they are ignored in highly air-entrained flows in stepped spillways, where they play significant roles, they could lead to scale effects and to wrong interpretations of results.

Boes (2000), Chanson (2002), Boes and Hager (2003a), and Takahashi et al. (2005) have published works on the impact of scale effects in modelling stepped spillways. Scale effects in stepped spillway models are likely to occur with scales lower than 10:1, Reynolds number smaller than 1 x 105, Weber number smaller than 100, and step heights lesser than 3 cm. They measured air-water flow properties at all step edges downstream of the inception point of air-entrainment with conductivity phase-detection intrusive probes, optical fiber probes. Use of a Dall Tube flow meter, or V-notch for flow rates, Prandtl-Pitot for flow velocities or Point gauge for clear water flow depth to obtain air-water flows properties is not practicable as large quantities of air are entrained at the air-water interface ( ).

The principle of the conductivity probe is based on the difference between resistivity of air and water, which provides an instantaneous voltage signal. Threshold technique analyze signals - from a single sensor - used to calculate a) the time-averaged local air concentration or void fraction C, b) the number of air-to-water (or water-to-air) voltage changes expressed as bubble count rate F, and c) the air bubble and water droplet chord sizes. For double-tip conductivity probe with longitudinal separation between the two probe sensors, the cross-correlation analysis of the signals leads to the local time-averaged interfacial velocity V. Details about the signal processing techniques can be found in [8] and [13].

The double-tip conductivity probes used had sensor sizes of Ø = 0.13 mm and 0.25 mm and were sampled for a period of 45s with a frequency of 20 kHz per sensor.

The discharges comprised transition and skimming flow rates 0.035 ≤ qw ≤ 0.234 m2/s for the spillways with θ = 8.9◦ and 0.02 ≤ qw ≤ 0.249 m2/s for θ = 26.6◦ comprising Reynolds numbers of 1.4 × 105 ≤ Re ≤ 9.3 × 105 and 8.1 × 104 ≤ Re ≤ 9.9 × 105 (Table 1).

Table 1 lists the channel slope, the step height h, the channel width W, the flow rate per unit width q_w, the flow rate dc/h with dc the critical flow depth.

Table 1.1: Summary of the 11 No Publications for the flat stepped spillway with chute angle of $3.4^o \leq \theta \leq 26.6^o$

Reference	Slope (θ)deg	Step geometry and Flow conditions	Instrumentation	N (No of Step)
Carosi and Chanson (2006)	21.8	h (cm) = 10 dc/h = 1.0 –1.57 qw (m^2/s)= 0.095-0.18 Re =$3.8x10^5$ – $7.2x10^5$	Double-tip (Ø=0.25mm) 20kHz/45s	10
Felder and Chanson (2009a)	21.8	h (cm) = 5 dc/h = 1.17 – 3.16 qw (m^2/s)= 0.059 - 0.10 Re =$2.4x10^5$ – $4.0x10^5$	Double-tip (Ø=0.25mm) 20kHz/45s	20
Guenther, Felder, and Chanson (2013)	26.6	h (cm) = 10 dc/h = 0.5 - 1.7 qw (m^2/s)= 0.03 - 0.217 Re =$1.2x10^5$ – $8.7x10^5$	Double-tip (Ø=0.025mm) 20kHz/45s	10
Toombes and Chanson (2005)	3.4	h (cm) = 14.3 dc/h = 0.61 - 0.92	N/A	10

		qw (m^2/s) = 0.08 - 0.15 Re =$3.2x10^5 - 6.0x10^5$		
Bung and Schlenkhoff (2009)	18.4 & 26.6	h (cm) = 3 & 6 dc/h = 2.65 - 3.58 qw (m^2/s)= 0.07 - 0.11 Re =$2.8x10^5 - 4.4x10^5$	N/A	2.4 m Hdam
Chanson and Toombes (2002b)	3.4	h (cm) = 7.15 & 11.3 dc/h = 0.61 - 1.85 qw (m^2/s) = 0.08 - 0.15 Re =$3.2x10^5 - 6.0x10^5$	Single-tip (Ø=0.35mm) 5kHz/60s & 180s	10 & 18
Ohtsu, Yasuda, and Takahashi (2004)	5.7 & 11.3	h (cm) = 0.63 & 5.0 dc/h = 1.25 - 14.3 qw (m^2/s)= 0.02 - 0.08 Re =$0.8x10^5 - 3.2x10^5$	Single-tip (Ø=0.1mm) 2kHz/60s	
Thorwarth (2008)	14.6	h (cm) = 5 & 10 dc/h =1.27 - 3.55 qw (m^2/s)= 0.05 - 0.234 Re =$2.0x10^5 - 9.4x10^5$	Double-tip (Ø=0.13mm) 30kHz/40s	26
Chanson and Toombes (2002a)	15.9 & 21.8	h (cm) = 10 dc/h = 0.78 - 1.53 qw (m^2/s)=0.069 – 0.188 Re =$2.8x10^5 - 7.5x10^5$	Double-tip (Ø=0.025mm) 20kHz/20s	9
Gonzalez (2005)	15.9	h (cm) = 5 & 10 dc/h = 0.6 - 3.2 qw (m^2/s)= 0.021 - 0.22 Re =$5.0x10^5 - 8.8x10^5$	Double-tip (Ø=0.025mm) 20kHz/20s	9 & 18
Bung (2009)	18.4 & 26.6	h (cm) = 3 & 6 dc/h = 2.65 – 3.58 qw (m^2/s)= 0.07 - 0.11 Re =$2.8x10^5 - 4.4x10^5$	Double-tip (Ø=0.13mm) 25kHz/25s	2.4 m Hdam
Gonzalez (2005)	21.8	h (cm) = 10 dc/h = 1.1 –1.7 qw (m^2/s)= 0.114 - 0.22 Re =$4.6x10^5 - 8.8x10^5$	Double-tip (Ø=0.025mm) 20kHz/20s	10
Felder and Chanson (2014)	8.9	h (cm) = 5 dc/h = 1.0 – 3.55 qw (m^2/s)=0.035 – 0.234 Re =$1.4x10^5 - 9.4x10^5$	Double-tip (Ø=0.13mm) 20kHz/45s	21
Felder and Chanson (2014)	26.6	h (cm) = 5 & 10 dc/h = 0.69 – 3.30 qw (m^2/s)= 0.02 – 0.227 Re =$0.8x10^5 - 9.1x10^5$	Double-tip (Ø=0.25mm) 20kHz/45s	10 & 20
Felder and Chanson (2014)	26.6	h (cm) = 10 dc/h = 0.82 – 1.85 qw(m^2/s)= 0.073 – 0.249 Re =$2.9x10^5 - 1.0x10^6$	Double-tip (Ø=0.25mm) 20kHz/45s	20

2.1 Formulation of the models

The authors obtained more than 700 data sets from the 11 No researchers (Table 1.). They reanalyzed about 500 with complete data to formulate energy dissipation models that govern transition and skimming flow over a wide range of operating conditions.

In modeling, it is necessary to determine the values of the parameters that can fit the model of the system it shall describe (Agunwamba, 2007). By the least square method, the best fit curve for this study was formulated as a function of the channel slope, the number of steps, the step height, and the critical water depth using multiple regression analysis and expressed here as:

$$\frac{\Delta H}{H_{max}} = \left[\alpha_o \frac{Nh}{y_c}\right]^{\alpha_1} N^{\alpha_2} h^{\alpha_2} \theta^{\alpha_3} \qquad [2.1]$$

Where

$\frac{\Delta H}{H_{max}}$ is the energy loss ratio,
H_{max} is the maximum available height,
N is the number of spillway steps,
h is the height of the spillway steps,
θ is the spillway channel slope.

They used the measured data sets and multiple regression analysis to solve Equation [2.1] which yielded the values of the constant αo along with the coefficients α1, α2, α3, and α4, which are then substituted in Equation [2.1] to give the developed models in 3.1.

3. RESULTS AND DISCUSSION

3.1 Developed models for skimming flow regimes.

3.1.1 Eq [3.1] is valid for use when h (cm) is not more than 20, N is not more 20, θ (degrees) is between 26.6° and 21.8°, and d_c/h is between 1.0 and 3.7.

$$\frac{\Delta H}{H_{max}} = \left[0.049 \frac{Nh}{d_c}\right]^{0.353} N^{0.06} h^{0.124} \theta^{-0.157} \quad [3.1]$$

3.1.2 Eq [3.2] is valid for use when h (cm) is not more than 20, N is not more 20, θ (degrees) is between 21.8° and 3.4°, and d_c/h is between 1.0 and 3.6.

$$\frac{\Delta H}{H_{max}} = \left[0.029 \frac{Nh}{d_c}\right]^{0.353} N^{0.06} h^{0.124} \theta^{-0.157} \quad [3.2]$$

3.2. Charts for 3.1.1

Figures 5 to 8 depicted the energy loss rates as a function of the expression of a dam height over critical depth for the measured data, the developed analytical formulation (Eq. [3.1]), the existing model for the computation of energy dissipation (Eq. [1.4]) with the friction factors of f = 0.30 and 1.30. The figures also show some traditional concave shape distributions for all the plotted four data sets for energy dissipation for all the flow rates. As seen in the charts, energy losses increased with decreasing discharges and increased with rising spillway height following earlier investigations (Matos, 2000; Chanson, 2001b; Felder & Chanson, 2009a). The measured data and the developed model data (Eq. [3.1]) are in close agreement with the coefficients of correlation from 0.96 to 0.99. Again, measured data are in close agreement with the existing model data (Eq. [1.4]) with a friction factor, f = 0.3, yielding the coefficients of correlation between 0.92 and 0.95.

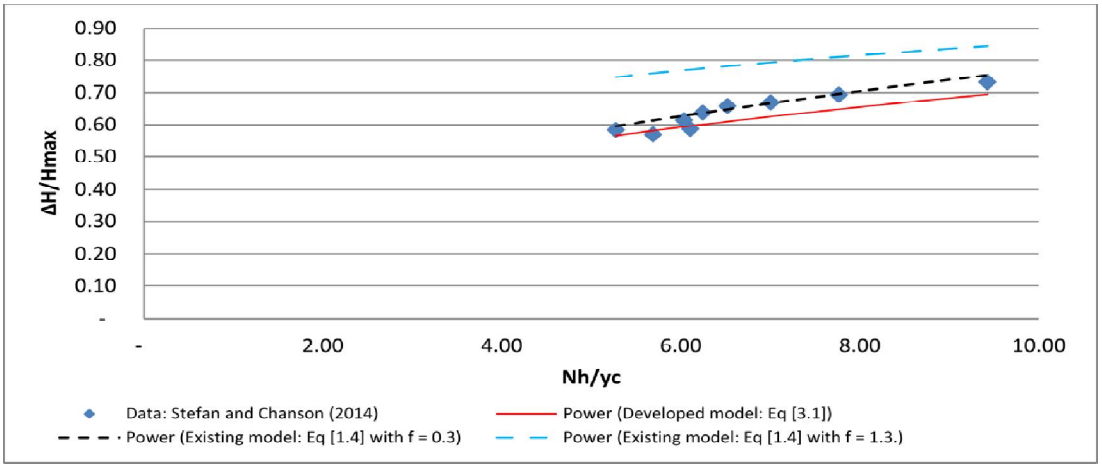

Fig. 5. $\Delta H/H_{max}$ as a function of Nh/d_c for θ = 26.6, N = 10, h= 10, q_w = (0.073 - 0.249) $m^2 s^{-1}$ & Re = (2.92 x 10^5 - 9.96 x 10^5), flow rate d_c/h of (0.82 - 1.85).

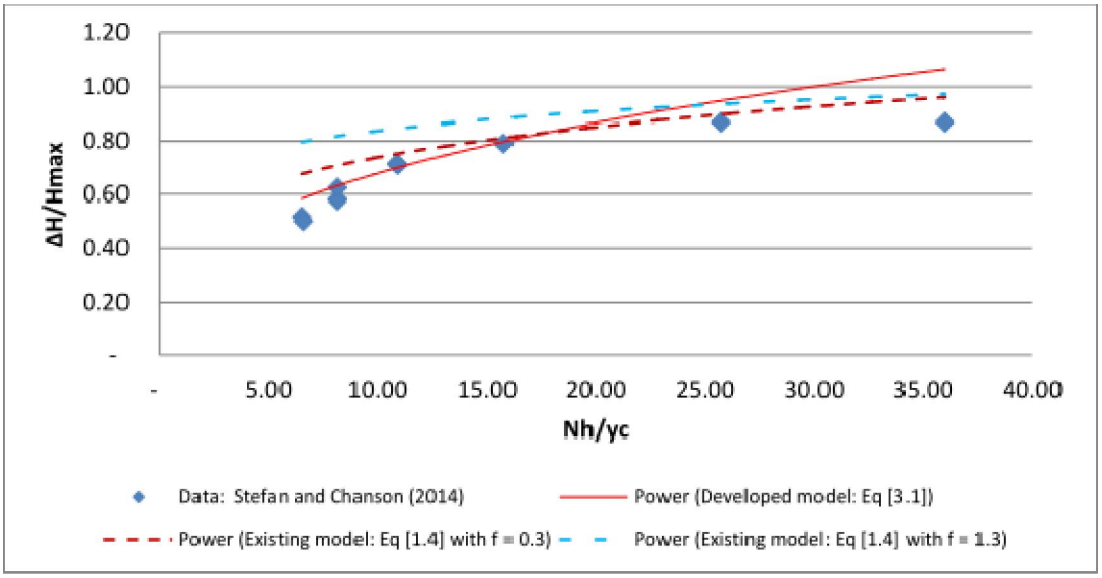

Fig. 6. $\Delta H/H_{max}$ as a function of Nh/d_c for θ = 26.6, N = 20, h = 5, q_w = (0.020 - 0.227) m^2 s^{-1} & Re = (8.0 x 10^4 - 9.08 x 10^5), flow rate d_c/h, of (0.69 - 3.30).]

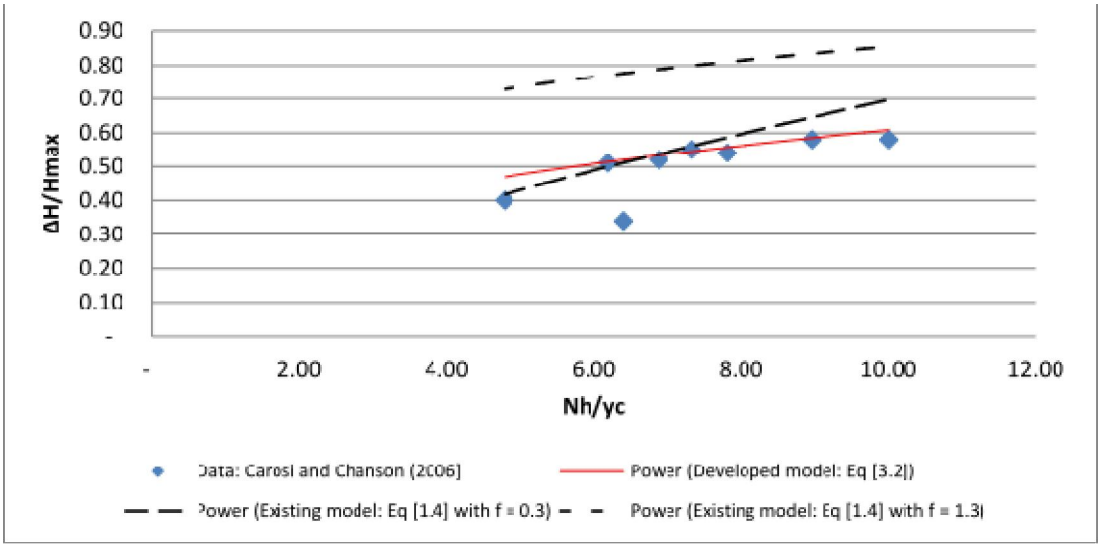

Fig. 7. $\Delta H/H_{max}$ as a function of Nh/d_c for θ = 21.8, N = 10, h = 10, q_w = (0.095 - 0.180) m^2 s^{-1}, Re = (3.80 x 10^5 - 7.20 x 10^5), flow rate, dc/h, of (1.00 - 1.57).

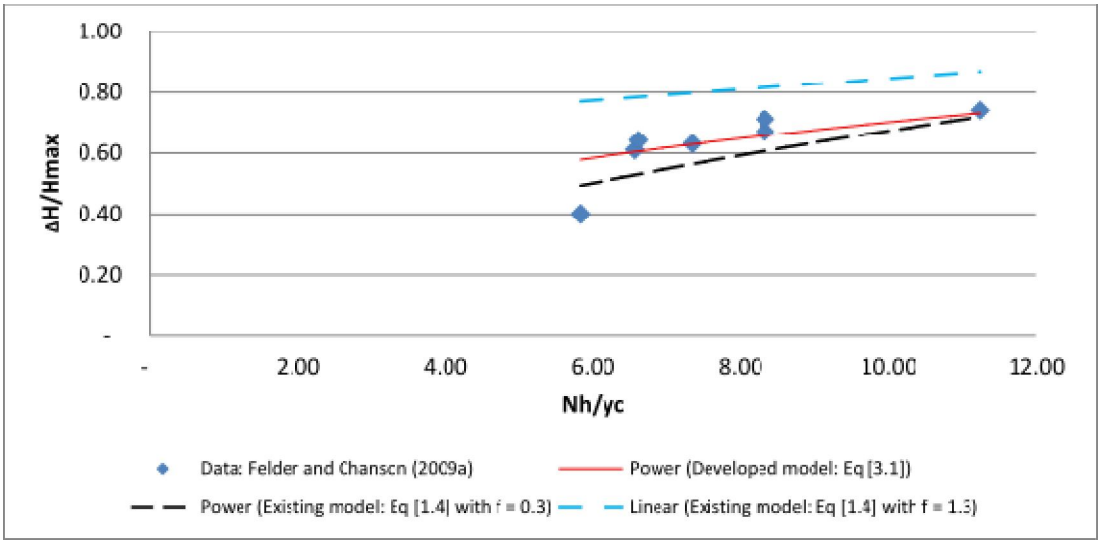

Fig. 8. $\Delta H/H_{max}$ as a function of Nh/d_c for θ = 21.8, N = 20, h = 5, q_w = (0.059 - 0.158) m^2 s^{-1}, Re = (2.36 x 10^5 - 6.32 x 10^5), flow rate, d_c/h, of (0.80 - 1.85).

3.3. Charts for 3.1.2

Figures 9 to 14 depicted energy loss rates as a function of a stepped spillway height over critical depth for the measured data, the developed analytical formulation (Eq. [3.2]), the existing model for the computation of energy dissipation (Eq. [1.4]) with the friction factors of f = 0.30 and 1.30. They displayed similar traditional concave shape distributions for all plotted four data sets for energy dissipation for all flow rates. As shown in the charts, energy losses increase with decreasing discharges and increase with rising dam heights that follow earlier investigations (Matos, 2000; Chanson, 2001b; Felder & Chanson, 2009a). They showed that measured data sets, the developed model data (Eq. [3.2]), and the existing model data (Eq. [1.4]) with friction factor, f = 0.3, are in close agreement with the coefficients of correlation from 0.95 to 0.99. Figure 9 showed that the developed model data (Eq [3.2]) was slightly higher than the measured data, increasing with increasing discharges. Figure 10 indicated that the developed model compared well with the measured data than the existing model with f = 0.10. Figures 11, 12, and 13 showed that the developed model compared well with field data sets. Figure 14 showed that the developed model predicted values slightly lower than the measured data, while the existing model with f = 0.10 produced data sets higher than the field data sets Figure 15 showed an interesting pattern: Rates of energy losses with a stepped spillway slope of 3.4o degrees reduced along the spillway with decreasing discharges and decreased with increasing stepped spillway height. Therefore, the need to investigate why this happened.

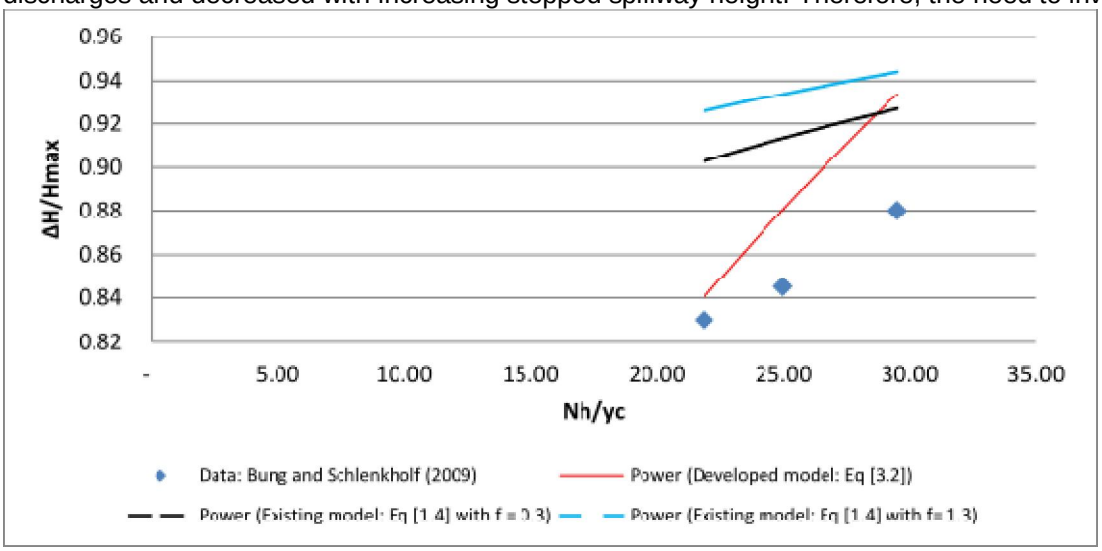

Fig. 9. $\Delta H/H_{max}$ as a function of Nh/d_c for θ = 18.4, N=40, h = 6, q_w = (0.059 - 0.158m) m^2 s^{-1}, Re = (2.36 x 10^5 - 6.32 x 10^5), flow rate, d_c/h, of (0.80 - 1.85).

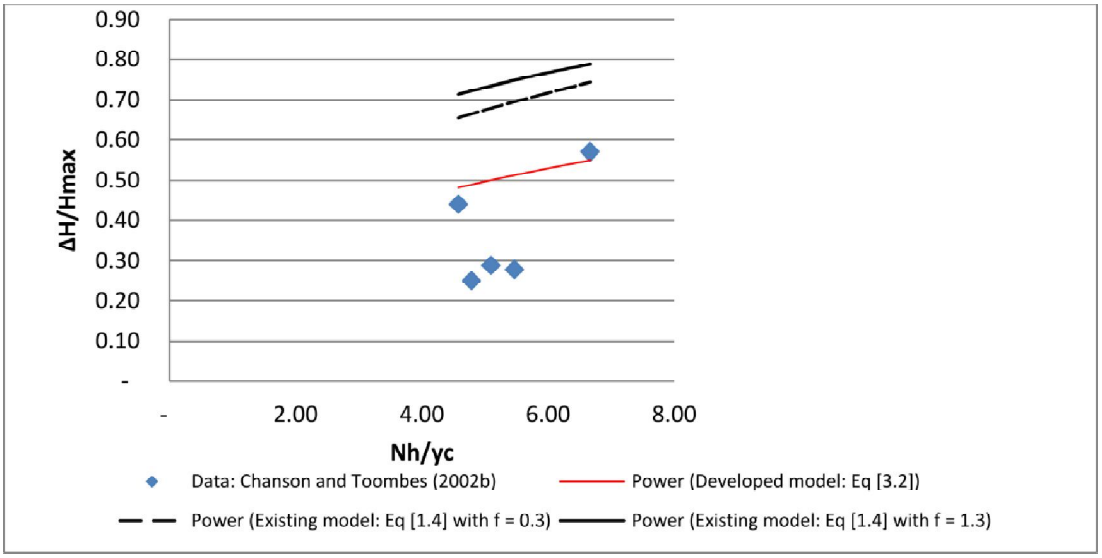

Fig. 10. $\Delta H/H_{max}$ as a function of Nh/d_c for θ =15.9, N = 9, h = 10, q_w = (0.069 - 0.188) m^2 s^{-1}, Re = (2.76 x 10^5 - 7.52 x 10^5), flow rate, dc/h, of (0.78 - 1.53)

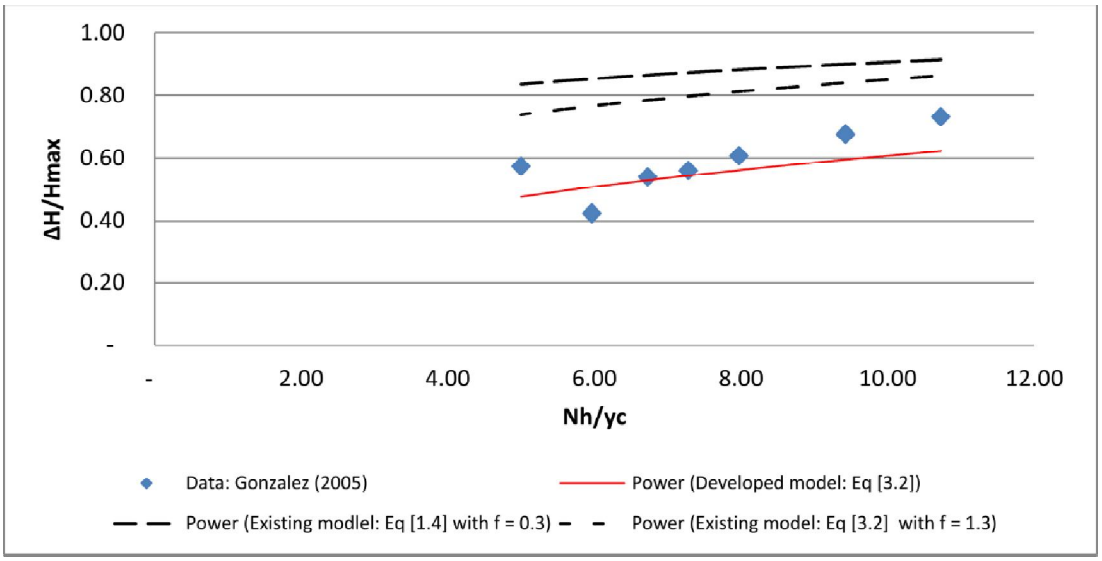

Fig. 11. $\Delta H/H_{max}$ as a function of Nh/d_c for θ = 15.9, N = 18, h = 5, q_w = (0.021 - 0.220) m^2 s^{-1}, Re = (8.4 x 10^3 - 8.8 x 10^5), & flow rate, dc/h of (0.60 - 3.20).

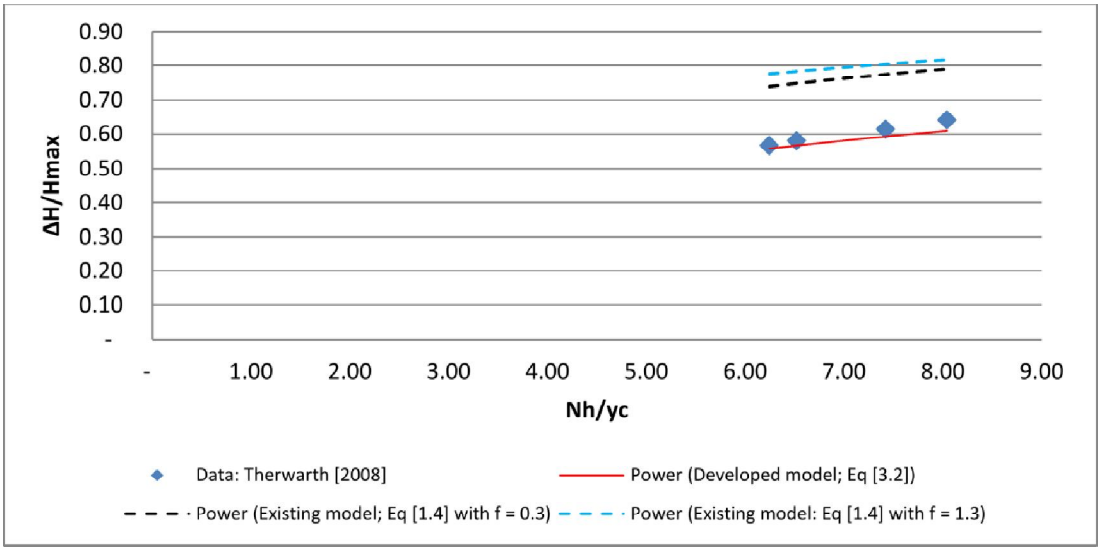

Fig. 12. $\Delta H/H_{max}$ as a function of Nh/d_c for θ = 14.6, N = 13, h = 10, q_w = (0.05 - 0.234) m^2 s^{-1}, Re = (2.0 x 10^5 - 9.36 x 10^5), & flow rate, dc/h, of (1.27 - 3.55).

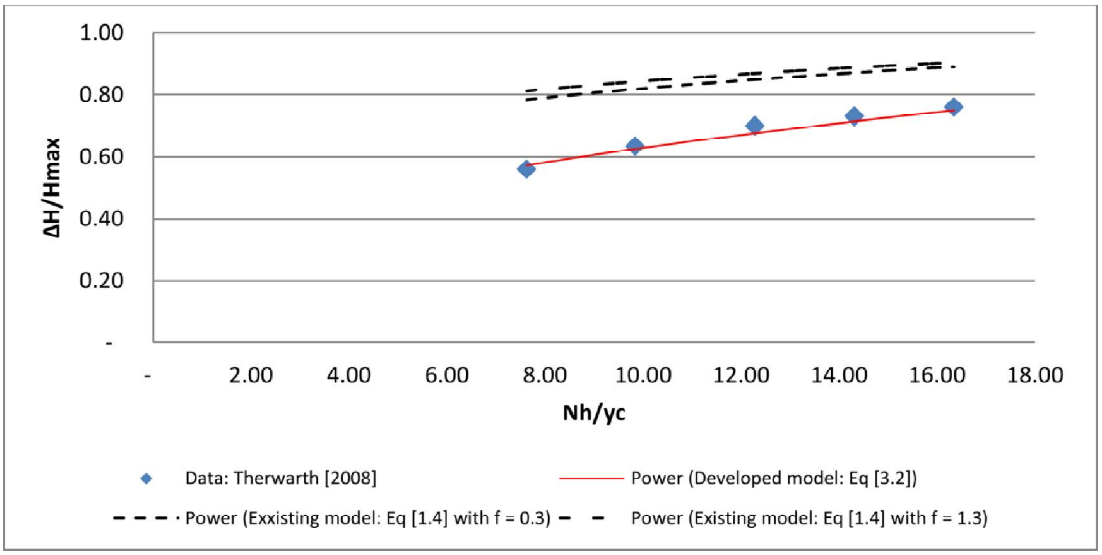

Fig. 13. $\Delta H/H_{max}$ as a function of Nh/d_c for θ = 14.6, N = 26, h = 10.0, q_w = (0.05 - 0.234) m^2 s^{-1}, Re = (2.0 x 10^5 - 9.36 x 10^5), & flow rate, d_c/h, of (1.27 - 3.55).

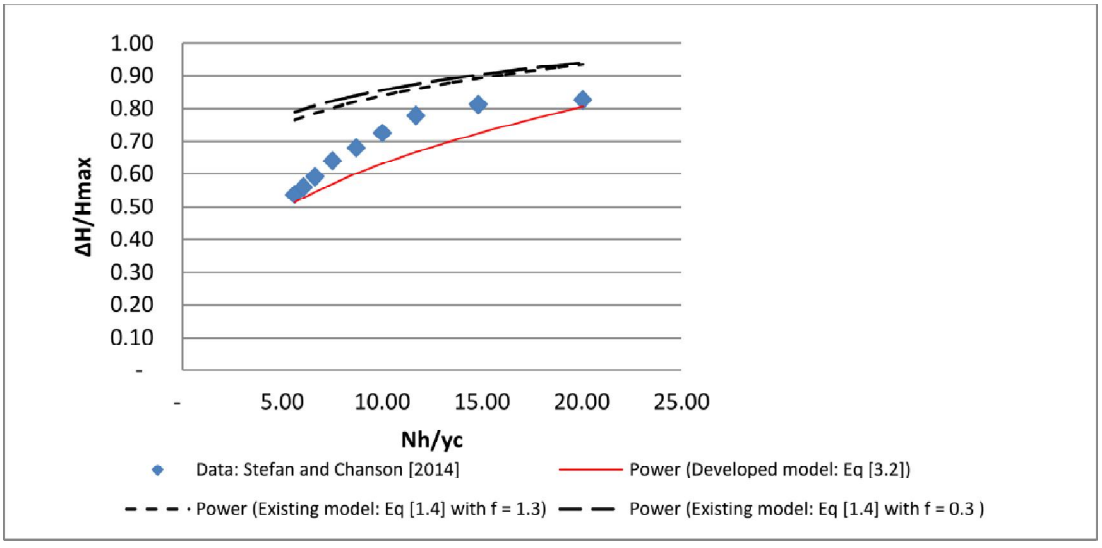

Fig. 14. $\Delta H/H_{max}$ as a function of Nh/d_c for θ = 8.9, N = 21, h = 3, q_w = (0.035 - 0.234) $m^2 s^{-1}$, Re = (1.40 x 10^5 - 9.36 x 10^5), & flow rate, dc/h, of (1.0 - 3.55).

Fig. 15. $\Delta H/H_{max}$ as a function of Nh/dc for q_w = (0.61 - 0.92m^2/s), Re = (2.44 x 10^6 - 3.68 x 10^6), & flow rate, dc/h, of 0.82 - 1.85.

4. CONCLUSION

Two new No-models for rates of energy losses in horizontal stepped spillways with slopes below 26.6^o degrees, with data based on detailed phase-detection probe measurements, were developed. They generated data compared well with the measured data sets. The new models also produced data sets that were in close agreement with the existing equation for rates of energy losses with friction factor, f = 0.10. Rates of energy losses increased along the stepped spillway with decreasing discharges and increased with increasing stepped spillway heights. However, rates of energy losses with a stepped spillway slope of 3.4^o
degrees reduced along the spillway with decreasing discharges and decreased with increasing stepped spillway height.

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COMPETING INTERESTS

The authors have no competing interests to declare.

AUTHORS' CONTRIBUTIONS

'Author 1' designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. 'Author 2' managed the literature searches. All the authors read and approved the final manuscript."

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LISTS OF SYMBOLS

d_c – critical water depth (m);
h - step height (m),
H_1 – residual head at the bottom of the spillway (m);
ΔH – difference between the maximum head and the residual head (m);
H - total head (m);
H_{max} - maximum head available (m):
$H_{max} = H_{dam} + 3/2 * d_c$;
Q - discharge ($m^2 s^{-1}$);
q_w - discharge per unit width ($m s^{-1}$);
Reynolds number defined as : $Re = \rho_w * U_w * D_H / \mu_w$
U_w - flow velocity (m/s): $U_w = q_w/d$;
W - channel width (m);

Subscript

c – conditions at critical height;
N– number of step;