Original Research Article

New equations for energy dissipation down a stepped spillway

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ABSTRACT

In this paper, the authors reviewed many previous publications on stepped spillways with a view to formulating new models for the rates of energy losses in stepped spillways. Stepped spillways are current method of choice for safe discharge of flood water due to their inherent ability to employ their stepping nature to safely dissipate substantial energy. Though many researchers have investigatedboththe hydraulic and geometric relationships that resulted in energy dissipation, quite a few of them had suggested models that could be used to estimate the rates of these energy losses. As a result, the authors, in this paper, obtained and examined more than 200 experimental data from the published works of the researchers like Carosi& Chanson (2006); Felder & Chanson (2009a); Guenther, Felder, & Chanson (2013); Toombes& Chanson (2005); Bung &Schlenkhoff (2009); Chanson &Toombes (2002b); Ohtsu, Yasuda & Takahashi (2004); Thorwarth (2008); Chanson &Toombes (2002a); Gonzalez (2005); Bung (2009); Gonzalez(2005); Stefan & Hubert Chanson (2014)on horizontal stepped channels slopes that ranged from 3.4 ≤ θ ≤ 26.6. They used about 150 of themthat had complete information, re-analyzed them, and used them to develop the energy dissipation models that governed skimming flows over a wide range of operating conditions. These models were later calibrated, which upon verification, gave good predictions between the measured and the computed data with high coefficients of correlation that ranged from 0.97 to 0.99, They are simple, easy to use, and render more accurate results than the existing model.

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Keywords:Aerated flow, energy dissipation, chute slope, dam height, stepped spillway, skimming flow.

1. INTRODUCTION

Stepped spillways are current method of choice for the safe discharge of flood water due to their inherent ability to employ their stepping nature to safely dissipate substantial energy. Significant damage may occur if the energy of the flow, especially its kinetic energy, is not dissipated safely. One type of flood release facility is the stepped spillway. It is characterized by significant flow resistance and associated energy dissipation caused by the steps. The design yields smaller, more economical dissipation structures at the downstream end of the chute. As the discharge down the chute is increased, a certain critical discharge would be reached beyond which air would be entrained in the flow. This phenomenon is known as a two phase flow or Air-Water flow. These Air-water flows on stepped spillways were investigated experimentally in the last few decades by researchers like Horner, 1969; Peyras et al, 1992; Chanson, 1995, 2001; Ohtsu& Yasuda, 1997; Chamani&Rajaratnam, 1999; Boes, 2000; Matos, 2001; Toombes, 2002. These researchers provided design guidelines for stepped spillway channels with various channel geometries and slopes. Many researchers have investigated both the hydraulic and the geometric relationships of stepped spillways of varying sizes that resulted in significant energy dissipation. These researchers provided design guidelines for stepped spillway channels with various channel geometries

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and slopes. Most recent research focused on the air-water flow properties and energy dissipation performances, including some more detailed air-water flow properties such as bubble count rate, turbulence, and microscopic air-water properties (Chanson &Toombes, 2002; Gonzalez, 2005; Chanson &Carosi, 2007; Toombes and Chanson, 2008a, b; Felder & Chanson, 2009b, 2011a); Meireles et al., 2009). However, despite these enormous amounts of information and design guidelines on stepped spillways provided by these scholars to design engineers, huge information gap and design guidelines still existed in the area of stepped spillways with channel slopes equal to or less than 26.6^{o}. It is, therefore, the purpose of this present study to obtain near-prototype scale data on the hydraulic characteristics of stepped spillway flow including air concentration, bulked flow depth, clear water depth, and flow velocity from the published works of the researchers like Carosi& Chanson (2006); Felder & Chanson (2009a); Guenther, Felder, & Chanson (2013); Toombes& Chanson (2005); Bung &Schlenkhoff (2009); Chanson &Toombes (2002b); Ohtsu, Yasuda & Takahashi (2004); Thorwarth (2008); Chanson &Toombes (2002a); Gonzalez (2005); Bung (2009); Gonzalez(2005); Stefan & Hubert Chanson (2014). These collected data were later re-analyzed to quantify energy dissipation and to develop design guidelines and 2 No models on stepped spillways with channel slopes equal to or less than 26.6^{o} that governed skimming flows over a wide range of operating conditions.

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These researchers were able to identify three kinds of flows that take place over a stepped spillway such as a) nappe flow regime, b) transition flow regime, and c) skimming flow regime.

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Innappe flow regime, sequence of drops from one step to the next step below it with the formation of hydraulic jump at every drop is observed. This type of flow can be likened to a sequence of separates drop structures (Chamani and Rajaratnam 1994; Chanson 1993).

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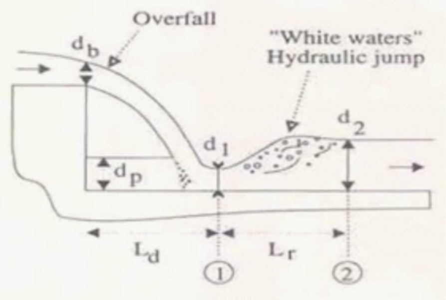

Figure 1: Nappe flow regime (Flow at a drop structure)

The water flows over one step of the spillway and lands on the next step with energy loss happening from a) the disintegration of the spout in the air, and b) the blendingof flowon the steps, with or withoutthe development ofhydraulic jump onthe step (Chanson, 1994; Rajaratnam, 1990) and these energy losses could be computed using equations [1.1] or [1.2].

$$\frac{\Delta H}{H_o} = 1 - \frac{\frac{d_1}{d_c} + \frac{1}{2}\left(\frac{d_c}{d_1}\right)^2}{H_{max} + \frac{3}{2}} \qquad [1.1]$$

Where d_1 is the water depth at impact, d_c is the critical water depth, and H_{max} is the dam height, ΔH is the energy loss, H_o is the maximum available energy, h is the height of the spillway step. Chanson (1994) later expressed this equation in terms of the spillway step height, the critical flow depth, and the dam height as:

$$\frac{\Delta H}{H_o} = 1 - \left[\frac{0.54 \left(\frac{d_c}{h}\right)^{0.275} + \frac{3.43}{2} \left(\frac{d_c}{h}\right)^{-0.55}}{\frac{3}{2} + \frac{H_{dam}}{d_c}} \right] \quad [1.2]$$

Nappe flow with completely established hydraulic jump (Figure 1), usually arises from small discharges with shallow flow depths and flow over the step with formation of supercritical at the edge of the step and returnsto subcriticalflow downstream of the jump.

In skimming flow regime, the flow occurs with the submergence of the steps with the development of fully aerated uniform flow in the downstream region in a long chute. Along the upstream steps, a non-aerated flow region exists within which a turbulent boundary layer develops. Air entrainment in the flow begins where the boundary layer intersects the free surface, referred to as the point of inception. Downstream from the point of inception, the flow continues to aerate and varies gradually in depth (Figure 2). The flow eventually becomes fully aerated, uniform flow in which the water depth, velocity, and air concentration become constant (Bindo et. al. 1993) (Figure 3).

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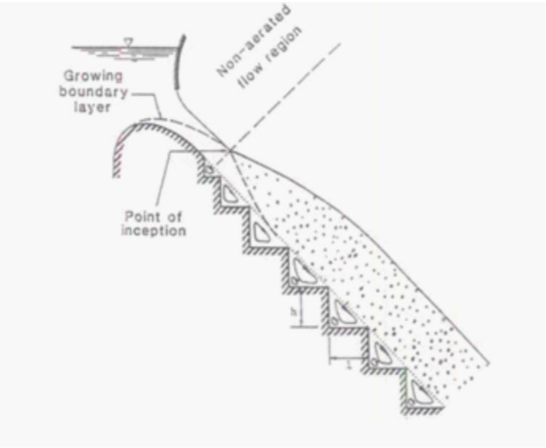

Fig. 2. Skimming flow regime - Sorensen (1985)

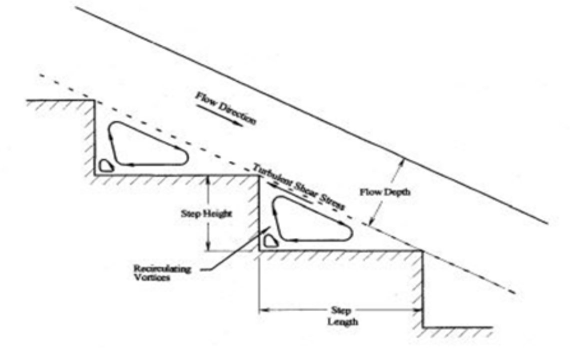

Fig. 3. Skimming flow regime with uniform flow conditions

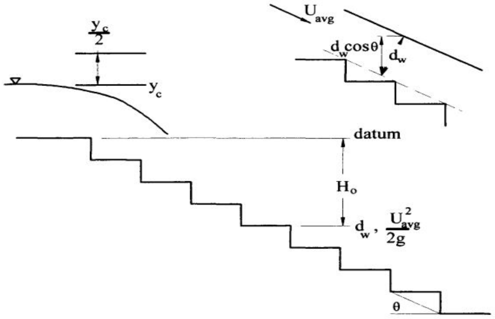

Fig. 4. Arrangement of the spillway with the definition of the variables

In skimming flow, most of the energy is dissipated in the maintenance of stable depression vortices. If uniform flow conditions are reached at the downstream end of the spillway, this energy loss could be computed as follows

$$\frac{\Delta H}{H_{max}} = \frac{d_w \cos\theta + \frac{U_{avg}^2}{2g}}{Nh + \frac{3}{2}d_c} \qquad [1.3]$$

Where d_w is the clear water depth, U_{avg} is the average velocity, the total head loss may be rewritten in terms of the friction factor, f, the spillway slope, θ, in degree, the critical depth,d_c, and the dam height,H_{dam}:

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\left(\frac{f}{8sin\theta}\right)^{1/3} cos\theta + \frac{E}{2}\left(\frac{f}{8sin\theta}\right)^{-2/3}}{\frac{H_{dam}}{d_c} + \frac{3}{2}} \qquad [1.4]$$

Eq [1.4] was computed for spillway slope with θ = 52 (degrees) and friction factor, f = 0.03 and f = 1.30, that represent average flow resistance on smooth spillways and stepped spillways, respectively. where E is the kinetic energy correction coefficient, ϑ is the dam slope in degrees.

2. MATERIAL AND METHODS

The authors obtained and re-analyzed the experimental data from the published works of Bung &Schlenkhoff (2009); Bung (2009);Carosi& Chanson (2006); Chanson &Toombes (2002b);Chanson &Toombes (2002a);Felder & Chanson (2009a); Guenther, Felder, & Chanson (2013); Gonzalez (2005); Gonzalez (2005);Ohtsu, Yasuda & Takahashi (2004); Stefan & Hubert Chanson (2014); Thorwarth (2008);Toombes& Chanson (2005);and used them to obtain them to develop 2No energy dissipation models that governed skimming flows over a wide range of operating conditions. These improved models were verified with the data sets obtained from the researchers and compared with the results from the existing model. These new models were then verified using part of the experimental data that were not used in the calibration. The extent of fit between the measured and the predicted data setswere statistically found and compared with the existing model.

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2.1 Formulation of the models

In modeling,it is necessary to determinethe values ofthe parametersthat can fit the model to the system it shall describe (Agunwamba, 2007). By the least square method, the best fit curve for this study was formulated as a function of the channel slope, number of steps, step height, and critical water depth using multiple regression analysis and expressed here as:

$$\frac{\Delta H}{H_{max}} = \left[\alpha_o \frac{Nh}{y_c}\right]^{\alpha_1} N^{\alpha_2} h^{\alpha_2} \theta^{\alpha_3} \qquad [2.1]$$

Where

$\frac{\Delta H}{H_{max}}$ is the energy loss ratio,
H_{max} is the maximum available height,
N is the number of spillway steps,
h is the height of the spillway steps,
θ is the spillway channel slope.

Using this equation and the measured data, the authors used multiple regression analysis and matrix method to solve Equation [2.1], which yielded the values of the constant $\alpha_o$along with the coefficientsα_1, α_2, α_3, and α_4, which were then substituted in Equation [2.1] to give the developed models in 3.1.

3. RESULTS AND DISCUSSION

3.1 Developed models for skimming flow regimes.

3.1.1 Eq [3.1] is valid for use when h (cm) is not more than 20, N is not more 20, θ (degrees) is between 26.6° and 21.8°, and d_c/h is between 1.0 and 3.7.

$$\frac{\Delta \mathrm{H}}{\mathrm{H_{max}}} = \left[0.049 \frac{\mathrm{Nh}}{\mathrm{d_c}}\right]^{0.353} \mathrm{N}^{0.06} \mathrm{h}^{0.124} \theta^{-0.157} \qquad [3.1]$$

3.1.2 Eq [3.2] is valid for use when h (cm) is not more than 20, N is not more 20, θ (degrees) is between 21.8° and 3.4°, and d_c/h is between 1.0 and 3.6.

$$\frac{\Delta \mathrm{H}}{\mathrm{H_{max}}} = \left[0.029 \frac{\mathrm{Nh}}{\mathrm{d_c}}\right]^{0.353} \mathrm{N}^{0.06} \mathrm{h}^{0.124} \theta^{-0.157} \qquad [3.2]$$

3.2. Charts for 3.1.1

Figures 5 to 8 depicted the energy loss rates as a function of the expression of a dam height divided by the critical depth for the measured data, the developed analytical formulation (Eq. [3.1]), the existing model for the computation of energy dissipation (Eq. [1.4]) with the friction factors of f = 0.30 and 1.30. These figures showed some traditional concave shape distributions for all the plotted four data set for energy dissipation for all the flow rates. As indicated in these charts, energy losses increased with decreasing discharges and again increased with increasing dam height for a particular discharge which is in accordance with the earlier investigations (Matos, 2000; Chanson, 2001b; Felder & Chanson, 2009a). The data sets from the field work of these researchers and the developed model (Eq. [3.1]) were in close agreement with the coefficients of correlation than ranged from 0.96 to 0.99. Again, these data sets from these field works were in close agreement with the existing model (Eq. [1.4]) when used with the friction factor, f, of 0.3, yielding the coefficients of correlation of between 0.92 and 0.95.

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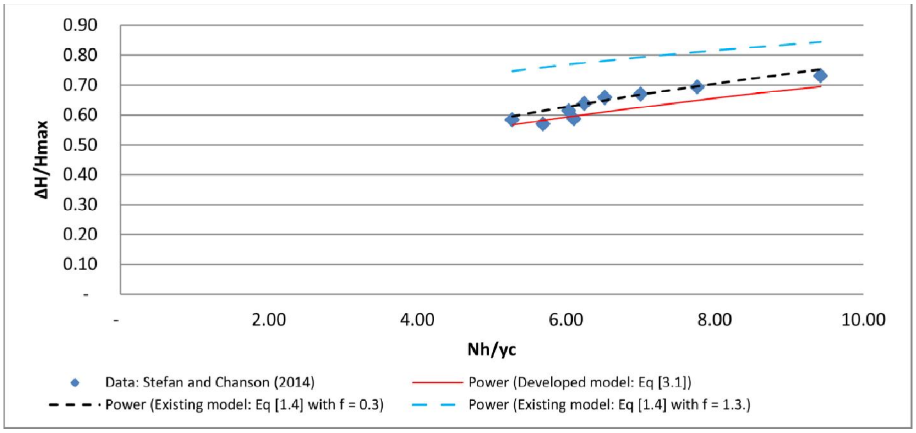

Fig.5.$\Delta H/H_{max}$ as a function of Nh/d_c for θ = 26.6, N = 10, h= 10, q_w = (0.073 - 0.249) m^2 s^{-1}& Re = (2.92 x 10^5 - 9.96 x 10^5), flow rate d_c/h of (0.82 - 1.85).

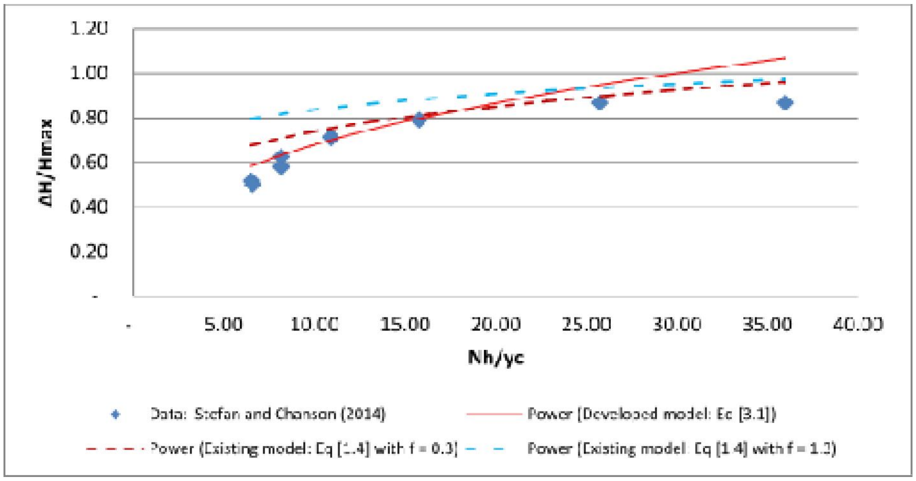

Fig. 6. ΔH/H_{max} as a function of Nh/d_c for θ = 26.6, N = 20, h = 5, q_w = (0.020 - 0.227) m^2 s^{-1}& Re = (8.0 x 10^4 - 9.08 x 10^5), flow rate d_c/h, of (0.69 - 3.30).]

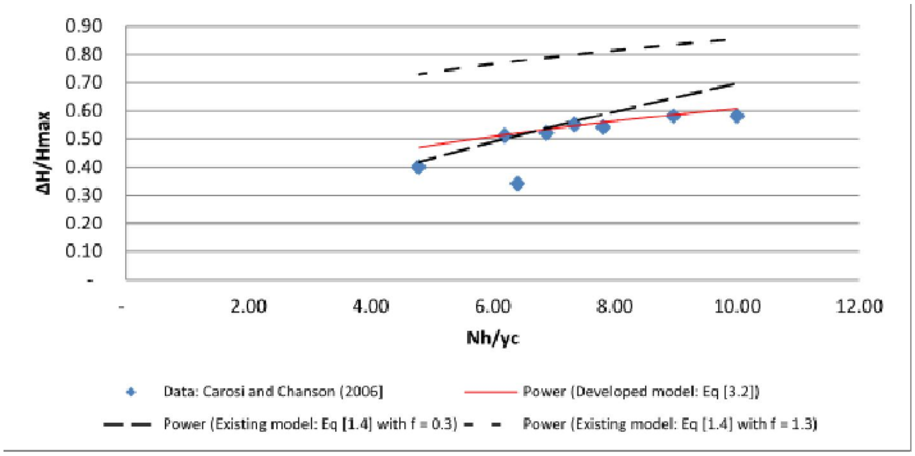

Fig. 7. ΔH/H_{max} as a function of Nh/d_c for θ = 21.8, N = 10, h = 10, q_w = (0.095 - 0.180) m^2 s^{-1}, Re = (3.80 x 10^5 - 7.20 x 10^5), flow rate, dc/h, of (1.00 - 1.57).

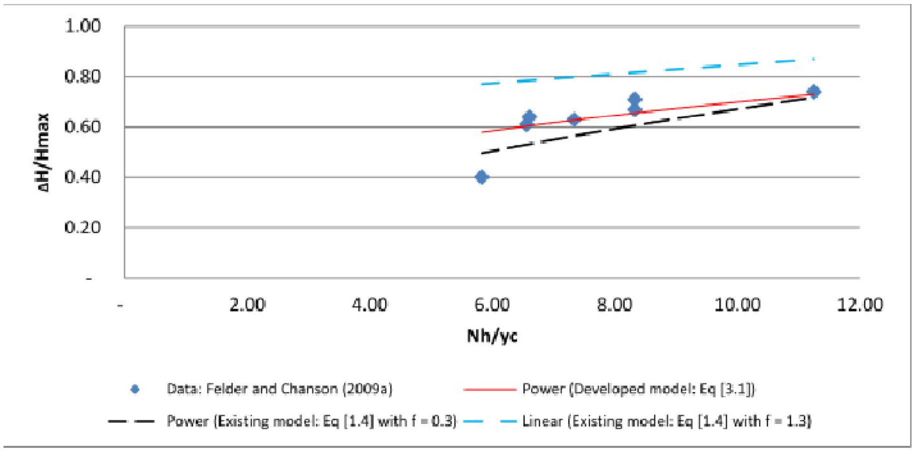

Fig. 8. $\Delta H/H_{max}$ as a function of Nh/d_c for θ = 21.8, N = 20, h = 5,q_w = (0.059 - 0.158) m^2 s^{-1}, Re = (2.36 x 10^5 - 6.32 x 10^5), flow rate, d_c/h, of (0.80 - 1.85).

3.3. Charts for 3.1.2

Figures 9 to 14 depicted the energy loss rates as a function of the expression of a dam height divided by the critical depth for the measured data, the developed analytical formulation (Eq. [3.2]), the existing model for the computation of energy dissipation (Eq. [1.4]) with f = 0.10, 0.30, and 1.3. The figures showed same traditional concave shape distributions for all the plotted four data set for energy dissipation for all the flow rates. As shown in the charts, energy losses increased with decreasing discharges and increased with increasing dam height for a particular discharge which is in accordance with the earlier investigations (Matos, 2000; Chanson, 2001b; Felder & Chanson, 2009a). The data sets from the field work, the developed model (Eq. [3.2]), and the existing model (Eq. [1.4]) when used with the friction factor, f = 0.3,were in close agreement with the coefficients of correlation than ranged from 0.95 to 0.99. Figure 9 showed that the developed model (Eq [3.2]) was slightly higher that the measured data, increasing with increasing discharges. Figure 10indicateeethat the developed model was in closer agreement with the measured data than the existing model with f = 0.10. Figure 11 showed that the developed model was in close agreement that with the measured data. Figures 12 and 13 showed that the developed model werein close agreement with the measured data. Figure 14 showed that the developed model predicted values that were slightly lower the measured data, while the existing model with f = 0.10 produced data that were slightly higher than the measured data.

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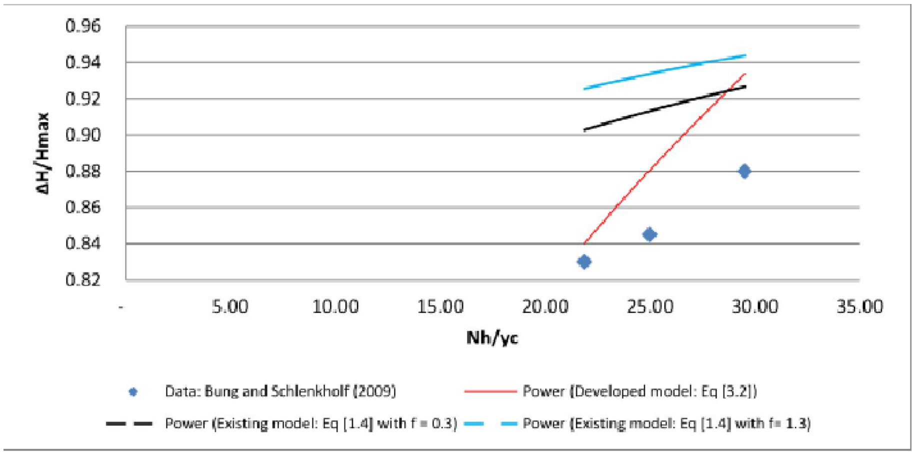

Fig. 9. $\Delta H/H_{max}$ as a function of Nh/d_c forθ= 18.4, N=40, h = 6,q_w = (0.059 - 0.158m) m^2 s^{-1}, Re = (2.36 x 10^5 - 6.32 x 10^5), flow rate, d_c/h, of (0.80 - 1.85).

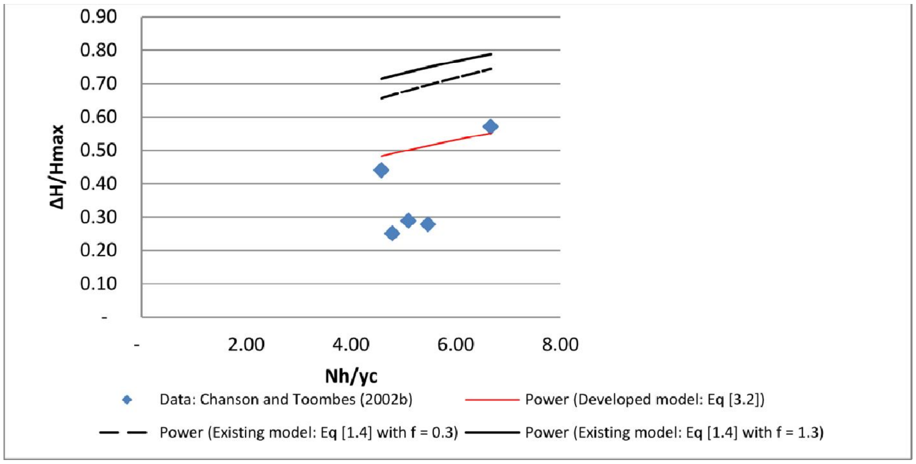

Fig. 10. $\Delta H/H_{max}$ as a function of Nh/d_c for θ =15.9, N = 9, h = 10, q_w = (0.069 - 0.188) m^2 s^{-1}, Re = (2.76 x 10^5 - 7.52 x 10^5), flow rate, dc/h, of (0.78 - 1.53)

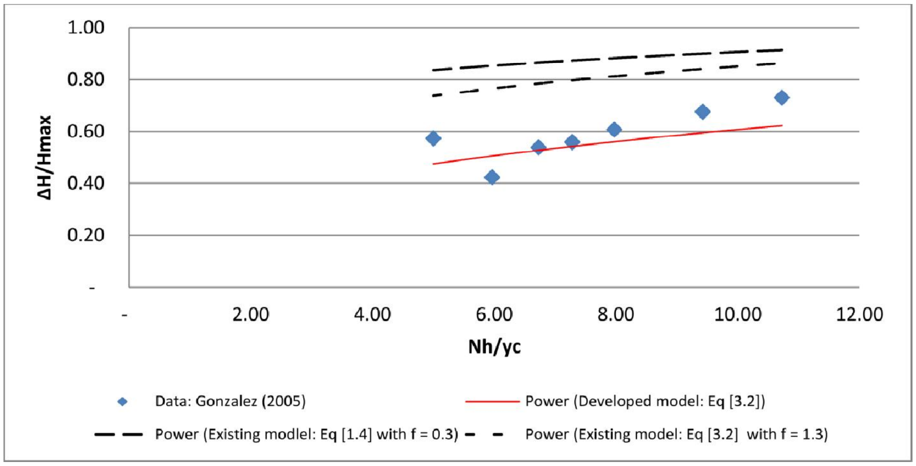

Fig. 11. $\Delta H/H_{max}$ as a function of Nh/d_c for θ = 15.9, N = 18, h = 5, q_w = (0.021 - 0.220) $m^2 s^{-1}$, Re = (8.4 x 10^3 - 8.8 x 10^5), & flow rate, dc/h of (0.60 - 3.20).

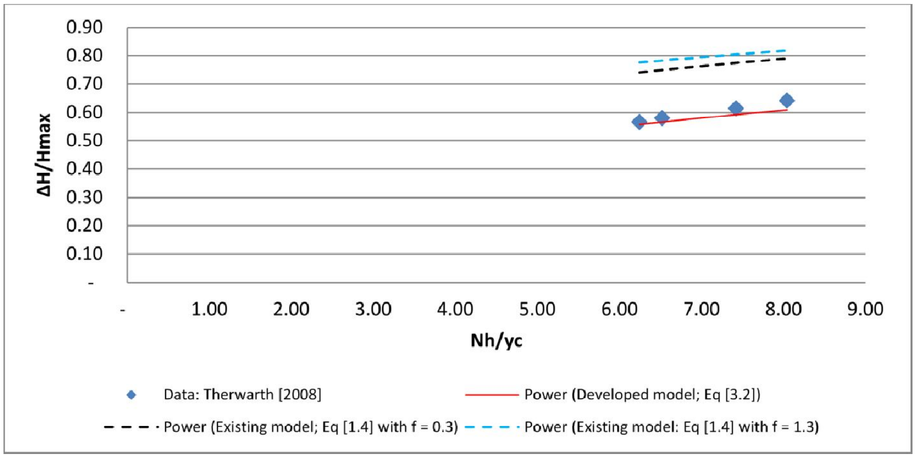

Fig. 12. $\Delta H/H_{max}$ as a function of Nh/d_c for θ= 14.6, N = 13, h = 10, q_w = (0.05 - 0.234) $m^2 s^{-1}$, Re = (2.0 x 10^5 - 9.36 x 10^5), & flow rate, dc/h, of (1.27 - 3.55).

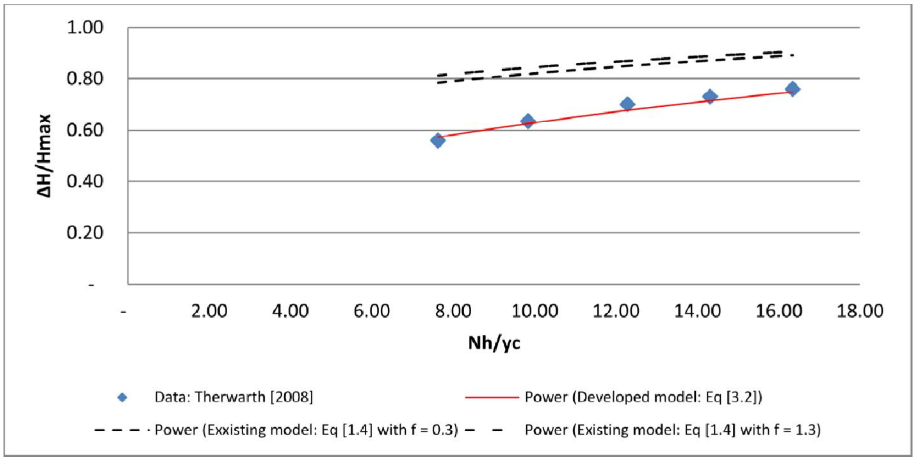

Fig. 13. $\Delta H/H_{max}$ as a function of Nh/d_c forθ= 14.6, N = 26, h = 10.0, q_w = (0.05 - 0.234) m^2 s^{-1}, Re = (2.0 x 10^5 - 9.36 x 10^5), & flow rate, d_c/h, of (1.27 - 3.55).

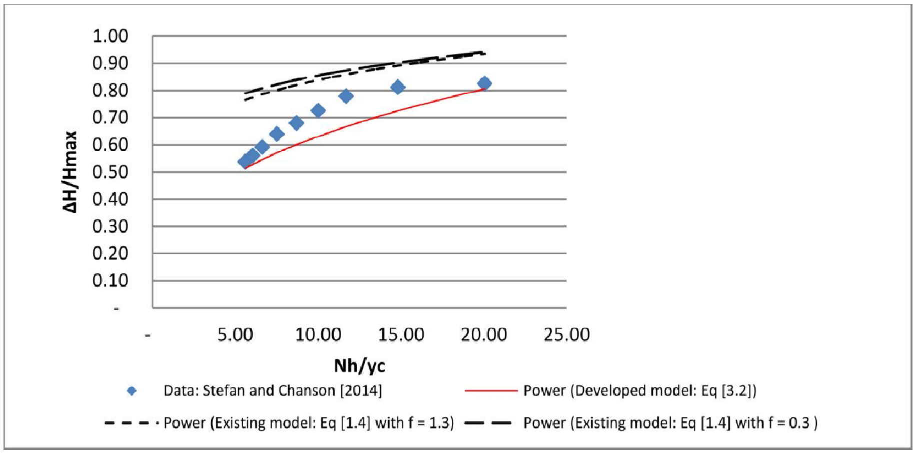

Fig. 14. $\Delta H/H_{max}$ as a function of Nh/d_c for θ= 8.9, N = 21, h = 3, q_w = (0.035 - 0.234) m^2 s^{-1}, Re = (1.40 x 10^5 - 9.36 x 10^5), & flow rate, dc/h, of (1.0 - 3.55).

4. CONCLUSION

Experimental data sets were obtained from the published works of 11No researchers' on eleven stepped spillway configurations with chute slopes between 3.4 (degrees) and 26.6 (degrees), step heights between 3 cm and 14.3 cm, unit discharges between 0.021 and 0.92 m2/m, Reynolds No between 0.84 x 105 and 36.8 x 105, discharge rate between 0.60 and 3.55 under skimming flow regime. These data re-analyzed, and used to develop 2No models that governedskimming flow regime in spillway under wide operating conditions. These new models were calibrated, verified and were found to be in close agreement with the measured data sets. It was also observed that the existing model for estimating energy losses when

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used with Darcy’s friction factors of 0.10 and 0.30, also yielded results that were in somewhat close agreement with the measured data sets.

Design engineers should use these new models for the design of stepped spillways as they provided simpler, easier to use equations that yielded better accuracy than that of the existing equation for the rate of energy dissipation in steeped spillways,

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LISTS OF SYMBOLS

d_c – critical water depth (m);
h - step height (m),
H_1 – residual head at the bottom of the spillway (m);
ΔH – difference between the maximum head and the residual head (m);
H - total head (m);
H_{max} - maximum head available (m):
$H_{max} = H_{dam} + 3/2 * d_c$;
Q - discharge ($m^2 s^{-1}$);
q_w - discharge per unit width ($m s^{-1}$);
Reynolds number defined as : $Re = \rho_w * U_w * D_H / \mu_w$
U_w - flow velocity (m/s): $U_w = q_w/d$;

W - channel width (m);

Subscript

c – conditions at critical height;
N– number of step;