Determination of energy dissipation in stepped spillway with fully developed hydraulic jump by a ball that bounced down through it.

(Okechuku Ozueigbo*[1], J. C. Agunwamba[2]

[1,2]Department of Civil Engineering, Faculty of Engineering, University of Nigeria, Nsukka

.

ABSTRACT

In this paper, the authors reviewed the characteristics of a ball that descended one step each bounce of a flight of stairs and drew an analogy with a stepped spillway with a fully developed hydraulic jump. They used this concept with equations of motion, classical hydraulic jump, and Chanson's model for energy dissipation in the stepped spillway to formulate a mechanistic model for energy dissipation rates. The range of Froude numbers used was between 5 and 16. They used data from previous researchers' publications on drop structures to verify the new model and data sets generated compared well with the measured data sets. Results showed that energy dissipation rates along the stepped spillway increased with the increasing number of steps but decreased with increasing discharges. It also showed the optimal number of steps for 100% energy dissipation.

Keywords: Energy Dissipation, Stepped spillway, Nappe flow, jet length, minimum chute length, hydraulic jump length

1.0 INTRODUCTION

In this paper, the authors reviewed the characteristics of nappe flow with a fully developed hydraulic jump on the stepped chute and drew an analogy with a ball that descended one step each bounce. They used the concept to develop a mechanistic model for the rates of energy dissipation for nappe flow with a hydraulic jump in a stepped spillway.

Nearly one-third of dams built in Europe and North America were equipped with a stepped cascade during the 19th century (Stephenson, 1991). More recently, the 1980s and 1990s were marked by renewed interest in stepped spillway design (Chanson, 2001).

Most structures were steep chutes for gravity dams operating in a skimming flow regime. For relatively low flow rates, the waters cascade down a stepped spillway as a succession of free-falling nappes.

Researchers like Chanson (1994) classified flows through stepped spillways into three regimes a) Nappe flow regime with a fully developed hydraulic jump, b) Nappe flow regime with a partial flow regime, and c) skimming flow regime. The nappe flow regime is a series of plunges from one step to another with the formation of nape at each drop. This type of flow is approximated by a series of single-step drop structures (Chamani and Rajaratnam, 1994; Chanson, 1993). The flow leaves the step as a free-falling jet and impinges on the tread of the next step. Energy dissipation occurs by the jet breakup, jet mixing on the steps, and formation of a partially or fully developed hydraulic jump on the steps (Chanson 1994; Rajaratnam 1990). Essery and Homer (1978) and Peyras et al.(1992) described a fully developed hydraulic jump (Fig. 1) as an isolated nappe flow. The flow passes through critical

depths at the brinks of the steps, forming supercritical free-falling jets, and returns to subcritical flow downstream of the jump.
In a nappe flow with a fully developed hydraulic jump the head loss at any intermediary step equals the step height. Hence, the total head loss, ΔH, along the spillway is the difference between the maximum head available, H_{max}, and the residual head, H_e, at the bottom of the spillway H_1 (Chanson, 1994).These energy losses could be calculated using equations [1] or [2].

For horizontal uniform steps, the flow depth at the edges is d_b= 0.715 * dc, where dc is the critical flow depth (Rouse, 1936).

Nappe flow with an established hydraulic jump (Fig. 1) usually occurs from small discharges with shallow flow depths. The flow cascades over steps, with the formation of supercritical flow at the edges of the steps, and returns to subcritical flow downstream of the jump.
In the nappe flow regime, a sequence of drops from one step to the next lower step occurs with the formation of a hydraulic jump on each step. This flow type is like a sequence of separate drop structures (Chamani and Rajaratnam 1994; Chanson 1993).
The water flows over one step of the spillway to the next lower step with energy loss occurring from: a) the disintegration of the jet in the air and b) the mixing of the flow on the steps, with or without the development of hydraulic jump, on the step (Chanson, 1994; Rajaratnam, 1990). Equations [1] and [2] can be adopted to calculate the rates of energy losses in stepped spillway.

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\frac{d_1}{d_c} + \frac{1}{2}\left(\frac{d_c}{d_1}\right)^2}{\frac{H_{max}}{d_c} + \frac{3}{2}} \qquad [1]$$

Where d_1 *is* the water depth at impact, d_c is the critical water depth, and H_{dam} is the dam height, ΔH is the energy loss, H_{max} is the maximum available energy, h is the height of the spillway step.
Chanson (1994) later expressed this equation in terms of the spillway step height, the critical flow depth and the dam height as:

$$\frac{\Delta H}{H_{max}} = 1 - \left[\frac{0.54\left(\frac{d_c}{h}\right)^{0.275} + \frac{3.43}{2}\left(\frac{d_c}{h}\right)^{-0.55}}{\frac{H_{max}}{d_c} + \frac{3}{2}}\right] \qquad [2]$$

Nappe flow with an established hydraulic jump (Fig. 1) usually arises from small discharges with shallow flow depths. It flows over the steps with the formation of supercritical flow at the edges of the steps and returns to subcritical flow downstream of the jump.

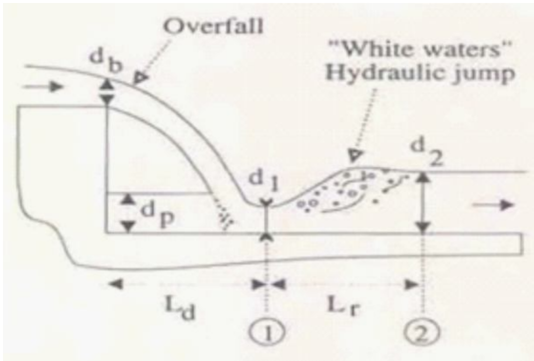

Fig. 1.Nappe flow trajectory at a fall structure

Estimate of the transition nappe flows on a stepped channel considering a nappe flows down in a single-step structure, the air cavity below the falling nappe will vanish when the circulating pool of water fills the total step cavity (Fig.1). The recirculating pool of water is essential as the accompanying pressure force offers a force parallel to the step surface, which is required to change the jet direction from an angle to a horizontal. For an aerated nappe, the momentum equation resolved along the step surface is:

$$\frac{1}{2}\rho_w g d_p^2 - \frac{1}{2}\rho_w g d_1^2 = \rho_w q_w (V_1 - V_i cos\vartheta_i) \qquad [3]$$

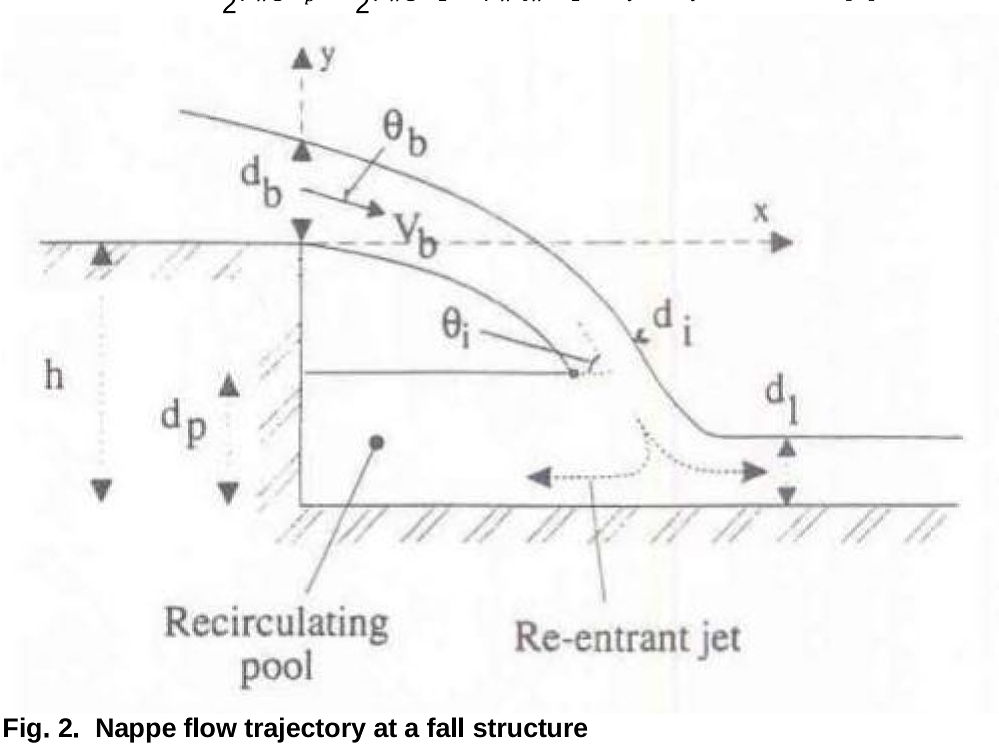

Fig. 2. Nappe flow trajectory at a fall structure

where
d_p = pool peak,
d_1 = flow depth downstream of the jet impact,
V_i = related velocity,
V_1 = impact velocity,

d_1= flow depth at the step edge,
V_b= flow velocity at the edge,
ϑi = jet impact angle.
ϑb = the original angle (Figure 2.6)
is the Froude number at the step brink of the streamlines with the horizontal is
ϑ_i = jet impact angle.

Equation [3] presumes that the brinks of the nappe have not broken down into spray, disregards the shear forces on the surfaces, and considers a hydrostatic pressure distribution at section 1 and the vertical face of the steps (Fig. 2).
The impact flow conditions can be inferred from simple trajectory calculations (Chanson, 1995):
where

$$\frac{d_i}{d_b} = \left(1 + \frac{1}{Fr_b^2}\right)^{-1/2} \quad [4]$$

$$\frac{V_i}{V_b} = \sqrt{1 + \frac{1}{Fr_b^2}} \quad [5]$$

$$\frac{cos\theta_i}{cos\theta_b} = \left(1 + \frac{1}{Fr_b}\right)^{-1/2} \quad [6]$$

Where
d_b= flow depth at the step edge,
v_b= flow velocity at the edge,
θ_b =original angle of the streamlines with the horizontal (Fig. 2),
F_{rb} =Froude number at the step brink.
At the jet crash on the step, and presuming that the velocity of flow into the control volume is the same as the one out of it, the momentum equation Eq. 2.1 yields:

$$\frac{d_p}{d_i} = \sqrt{1 + \frac{2v_i^2}{gd_i}(-\cos\theta_i)} [7]$$

White (1943) and Rouse (1955) analyzed a unit-step drop structure (Fig 2). For a flat step, the flow situations close to the steps change from the sub-critical flow to critical flow at a definite sector upstream of the brink (Chanson, 1994)
Where d_b, is given by ROUSE (1936) as

$$d_b = 0.715 d_c [8]$$

where
d_c = critical flow depth.
Downstream of the brink of the step, Rand (1955) linked the following nappe geometry

$$\frac{d_1}{h} = 0.54\left(\frac{d_c}{h}\right)^{1.275} \quad [9]$$

$$\frac{d_2}{h} = 1.66\left(\frac{d_c}{h}\right)^{0.81} \quad [10]$$

$$\frac{d_p}{h} = \left(\frac{d_c}{h}\right)^{0.66} [11]$$

$$\frac{L_d}{h} = 4.30\left(\frac{d_c}{h}\right)^{0.81} [12]$$

Resulting to

$$\frac{d_1}{d_c} = \frac{2^{\frac{1}{2}}}{\frac{3}{2^{\frac{3}{2}}} + \sqrt{\frac{3}{2} + \frac{h}{d_c}}} White(1955)[13]$$

where d_1, d_2 , d_p , d_c , L_d , L_r , h are the flow depth at sector 1, flow depth at sector 2, are the distance from the vertical façade of the step to the point of contact, the roller length of a completely established hydraulic jump, and length of a flat step, respectively (Fig. 1) (Chanson, 1994).

The flow depth, d_2, and the entire head at the sector 2 (Fig.1) are linked by the traditional hydraulic jump equation (Chanson)

$$\frac{d_2}{d_1} = \frac{1}{2}\left\{\sqrt{1 + 8Fr_1^2} - 1\right\} [14]$$

Where L_r, d_1, and Fr_1 are the roller length of a completely established hydraulic jump,
The depth of flow and the Froude Number respectively directly upstream of the jump is given as;

$$Fr_1 = \frac{q_w}{\sqrt{gd_1}} [15]$$

1.2 Conditions for nappe flow regime

If the length of the descent plus the length of the roller, Lr, is smaller than the length of the flat step, l, an established hydraulic jumpoccurs (Fig 1) Chanson (1994). Merging equations (9) and (10) a situation for an established napped flow is obtained: a nappe flow regime with a hydraulic jump takes place for flow rates that are less than a critical value defined by Chanson (1994) as:

$$\left(\frac{d_c}{h}\right)_{char} = 0.0916\left(\frac{h}{l}\right)^{-1.276} [16]$$

Where the relationship (Eq[16] was found for $0.2 \leq {}^{h}/_{l} \leq 6$
And l is the step length. Nappe flow for completely established hydraulic jump takes place for

$${}^{d_c}/_{h} < \left({}^{d_c}/_{h}\right)_{char} \qquad [17]$$

Stephenson (1991) suggested from the knowledge gained from dams with stepped spillways in South Africa that the suitable condition for nappe flow settings is:

$$\tan\theta = {}^{h}/_{l} < 0.2 \qquad [18]$$

For nappe flow to take place, the step flat length should be more than the water depth (Stephenson 1991, Lejeune et al. 1994)

1.3 Trajectory of a free falling ball

A ball released from a certain height will take equal time to drop as another ball projected horizontally from the same high, notwithstanding the different path length, as the vertical acceleration is the same. A ball thrown vertically up at twice the speed will rise four times higher, not because of v2 = 2as (an equation rather than an "explanation"), but because it takes twice as long to stop and travels twice as fast on average. A ball thrown obliquely onto the floor without spin will bounce with spin due to the horizontal friction force on the ball. A smaller ball will spin faster than a big ball of similar mass as it has a small moment of inertia. When a falling ball first contacts a rigid horizontal surface, the bottom of the ball instantly comes to a complete stop. All balls are sufficiently flexible that the top of the ball will continue to fall even after the bottom comes to rest. The ball will therefore compress like a spring, progressively from the bottom end up as each new section comes to rest. However, the bottom end of a ball is much softer than the middle section since the cross-sectional area is smaller, being a few mm in diameter at first compression. The effect is equivalent to having a relatively stiff and heavy mass compressing down on a relatively light and soft spring. The result is that only the bottom part of a ball compresses in a low-speed collision. A large portion of the ball compresses in a high-speed impact (Rod, 2015).

Trajectory refers to the path a falling or a rising object follows. In projectile, only acceleration due to gravity (g), which acts in the vertical direction, is considered in the equation of motion. There is no acceleration in the horizontal direction, with the small air resistance ignored in the equation of motion. Some examples of Projectile Motion are Football, A baseball, A cricket ball, or any other object. The projectile motion consists of two parts: one in the horizontal direction of zero acceleration and the other in the vertical direction of constant acceleration due to gravity. The projectile motion is always in the form of a parabolaand represented as:

$$y = ax + bx^2 \qquad [19]$$

Damping of motion occurs when air resistance is neglected and calculated as the square root of the ratios of maximum height after and before a bounce. The velocity of a ball with mass, m, which falls freely from a height, h1, and hits the ground, can be given as

$$v_1 = \sqrt{2gh_1} \qquad [20]$$

g = the acceleration due to gravity.

This is true because the potential energy, mgh_1, is completely change into kinetic energy, $mv_1^2/2$. The velocity v_2 of the ball upon bounces to a height, h_2, is given as

$$v_2 = ev_1, \qquad [21]$$

Where e is the coefficient of restitution and v_2 is derived as

$$v_2 = \sqrt{2gh_2} \qquad [22].$$

Therefore, $$e = v_2/v_1 = \sqrt{h_2/h_1} \qquad [23]$$

The time, t_1 for the first fall, h_1, is

$$h_1 = ut_1 + gt_1^2/2 \quad [24]$$

Where u = the initial vertical velocity = 0

$$\text{Simplifying, } t_1 = \sqrt{2h_1/g} \quad [25]$$

Similarly, the time, t_2, for the second fall, h_2, is

$$t_2 = \sqrt{2h_2/g} = et_1 \quad [26]$$

The total time, T, from the initial time to complete rest is therefore given as

$$T = t_1 + 2t_2 + 2t_3 + \dots = t_1 + 2et_1(1 + e + e^2 + \dots)$$
$$= t_1 + 2et_1/(1 - e) = t_1(1 + e)/(1 - e)$$

$$T = \sqrt{2h_1/g}\,(1 + e)/(1 - e) \quad [27]$$

The point to note here is that most mathematical models predict ball bounces as being infinitely, where the total time for the ball to come to rest is finite (Heck and Ellermeijer, 2009).

The authors studied the trajectory of the ball that bounced down the stairs with energy loss occurring at bounce. They assumed that the ball bounces off each step and hit the steps below as a free-falling object with energy dissipation occurring on collision with the horizontal step surface.
For simplicity, they assumed that the coefficient of restitution (e) was constant during the entire bounce dynamics and took on a positive value 'e' less than unity. They considered a stairway, which consisted of horizontal and vertical parts only.

1.4 Empirical modeling/Mechanistic modeling.

Although many researchers have investigated the hydraulic performance of stepped spillways, significant information gaps still exist in the guidelines for the design of stepped spillways. Most of them proposed empirical models for rates of energy loss in stepped spillways, which in most cases are applicable only for the modeled experiment under a particular operating condition. Therefore, empirical models cannot predict beyond certain operating ranges or designed experiment: Hence, the need for mechanistic models.
Mechanistic Modeling is adopted where the physical occurrence is approximated by considering the best essential processes and neglecting other less important effects that can complicate the problem but do not add considerably to the accuracy of the solution. It should be sufficiently close to the natural phenomenon as the flow pattern involved should not be overlooked. Yet, it should be largely simple so that the solution is amenable to reasonable analytical or numerical efforts (Taitel, 1994).

2. METHODOLOGY

The authors used the Newton Law of Motion, the classical hydraulic jump equation (Eq [5]), and the energy dissipation at nappe flow equation (Eq [9]) to develop this model. They used the law of motion to formulate the trajectory of an elastic pinball that bounced down a drop structure of horizontal step, l, and height, h, (Fig 1) and later extended the concept to a flight of stairs made up of multiples of similar drop structures.

They assumed

i) That the ball descended one step each bounce
ii) That after each bounce, it rebounded to a height, d_2,
iii) That the height, d_2, was large enough compared with the width of the step so that the impacts were effectively head-on;
i) That the coefficient of restitution, e, is constant and is less than 1;
ii) That air resistance is overlooked;
iii) That the only acceleration in the system is the acceleration due to gravity, g;
iv) That there is no acceleration in the horizontal direction,
v) that the trajectory of a freely falling ball is similar to that of the nappe flow regime, which is distinguished by a series of plunges, from one step to another with the formation of a nappe at each drop;
vi) that ball passed through a critical depth at the brink of each step, forming a supercritical free-falling flow and returned to subcritical flow downstream of the jump and;
vii) that the manner energy is dispersed in a nappe flow regime with a full hydraulic jump is similar to the way energy is lost with a freely falling ball descended one step with each bounce.

Since the collision occurred with the vertical component of the incidence velocity,the rebounded vertical velocity became ‘e’ times the opposite of this bound vertical velocity while the horizontal distance the ball traveled from d_1 to d_2 or x, became ‘ut’, measured from the edge of the step where bounce occurred.

The ball bounced off the step tread through the height,

$$d_2 - d_1 \quad [28]$$

And landed on the next lower step through the height,

$$d_b + h - d_1 [29]$$

where d_b, h, d_1, and d_2, respectively are the flow depth at the brink of the horizontal step, the height of step, the depth of bounce at section 1, and the depth of bounce at section 2 (Fig 1).

The coefficient of restitution denoted by ‘e’ - and ranges between 0 and 1 - is the ratio of the final to initial relative velocity between two objects at collusion is defined as

$$e = \sqrt{\frac{(d_2 - d_1)}{(d_b + h - d_1)}} \quad [30]$$

2.1 Measured data sets for verification of the new model

They obtained more than fifty field data from the published works on the rates of energy dissipation of drop structures by Moore (1943), Rand (1955), and Stephenson (1979) and used them to verify the new model.

2.2Formulation of model

Simplifying, Eq [3] yields

$$d_1 = \frac{(d_2 - e^2 d_b - e^2 h)}{(1 - e^2)} [31]$$

Substituting *equa*tions [8] and [31] into equation [31] and simplifying yields

$$d_1 = \frac{\left(\frac{d_1}{2}\left\{\sqrt{1 + 8Fr_1^2} - 1\right\} - e^2 0.75 d_c - e^2 h\right)}{(1 - e^2)(e^2 - 1)} \qquad [32]$$

$$d_1 = \frac{-e^2(0.715 d_c + h)}{(1 - e^2)(e^2 - 1) - 0.5\left(\sqrt{1 + 8Fr_1^2} - 1\right)} [33]$$

Dividing Eq [33] by d_c, gives

$$\frac{d_1}{d_c} = \frac{-e^2\left({0.715 d_c}/{d_c} + {h}/{d_c}\right)}{(1 - e^2) - 0.5\left(\sqrt{1 + 8Fr_1^2} - 1\right)} [34]$$

and finally substituting Eq [34] in Eq [1], yields

$$\frac{\Delta H}{H_{max}} = 1 - \frac{A + B}{\frac{3}{2} + \frac{Nh}{d_c}} [35]$$

where

$$A = \frac{-e^2\left(0.715 + {h}/{d_c}\right)}{(1 - e^2) - 0.5\left(\sqrt{1 + 8Fr_1^2} - 1\right)}$$

$$B = \frac{1}{2}\left(\frac{(1 - e^2) - 0.5\left(\sqrt{1 + 8Fr_1^2} - 1\right)}{e^2\left(0.715 + {h}/{d_c}\right)}\right)^2$$

and N = the number of steps.

3. RESULTS AND DISCUSSION

3.1: Determination of the appropriate values of the Coefficients of Restitution (e) and the Froude No (Fr_1)

a) The values of e = 0.95 and Fr_1 = 6.0 substituted in Eq [35] compared well with the measured data from Moore (1943) and were selected from Fig 3 to produce Fig. 4.

b) The values of e = 0.70 and Fr_1 = 16.0 substituted in Eq [35] compared well with the measured data from Rand (1955) and were selected from Fig 5 to produce Fig. 6

c) The values of e = 0.90 and Fr_1 = 5.0 substituted in Eq [35] compared well with the measured data from Stephenson (1979) and were from Fig 7. These values were then substituted in Eq [35] to produce Fig. 8.

Figures 3 to 9 showed that all the plotted curves followed the traditional concave shape distributions for rates of energy dissipation for all the flow rates (Chanson, 2001). They also showed that the rates of energy losses increased with an increasing number of steps and decreased with increasing discharges, which agreed with the previous researchers (Matos, 2000; Chanson, 2001b; Felder & Chanson, 2009a).

Fig 3 depicted energy losses rates against flow rates plotted with the data from Moore (1943), the data sets from Eq [35] with the parameters of e = 0.95, Fr_1 = 5.0, N = 1 as well as the data sets from Eq [35] with the parameters of e = 0.95, Fr_1 = 6.0, N = 1. As shown in the chart, the measured data sets and the data sets from Eq [35] with e = 0.95, Fr_1 = 6.0, and N = 1 were in close agreement. These parameters were, therefore, adopted for use in Eq [35] to produce the charts in Figures 2 and 8.

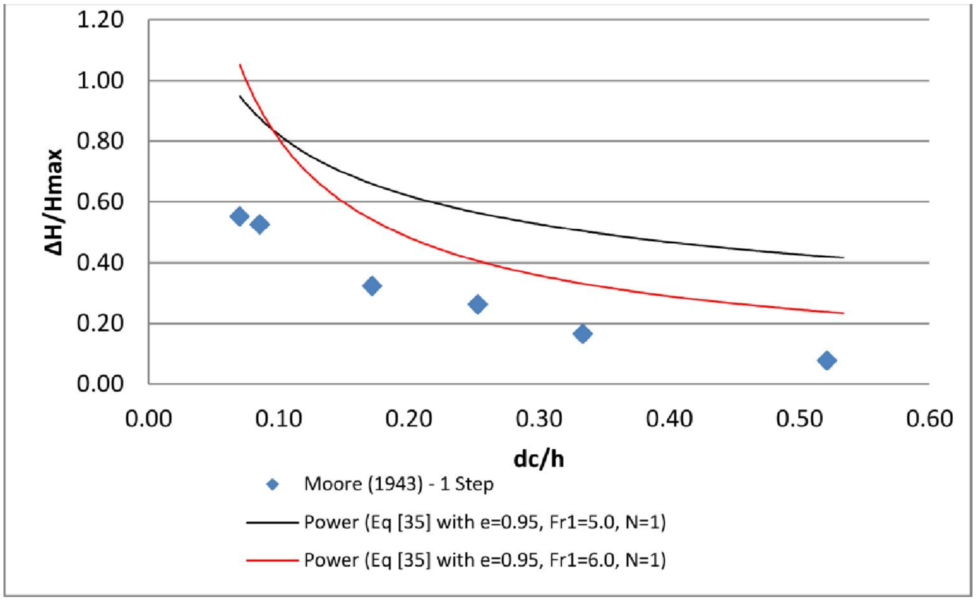

Fig.3. Variation of relative head loss, $\Delta H/Hmax$,with d_c/h(= 0.07 – 0.52) for e = 0.95, 5.0 < $Fr_1 \leq$ 6.0, N = 1

Fig 4 depicted energy losses rates against flow rates plotted with the data sets from Moore (1943) with N = 1, the data sets from Eq [35] with e = 0.95, Fr_1 = 6.0, and N = 1, 5, 10, 15, and 20.
The new model grossly overestimated rates of energy losses for values of d_c/h less than 0.20 and, therefore, should be used with caution for these values. The rates of these overestimation decrease with increasing flow rate, d_c/h. The model, however, predicted energy dissipation rates that compared fairly well with values of d_c/h between 0.20 and 0.53.
The chart showed that about 100% energy dissipation rates were achieved with about number of steps, N = 99. Hence, 99 is the optimal number of steps needed to achieve about 100% rates of energy dissipation with values of dc/h between 0.20 and 0.53.

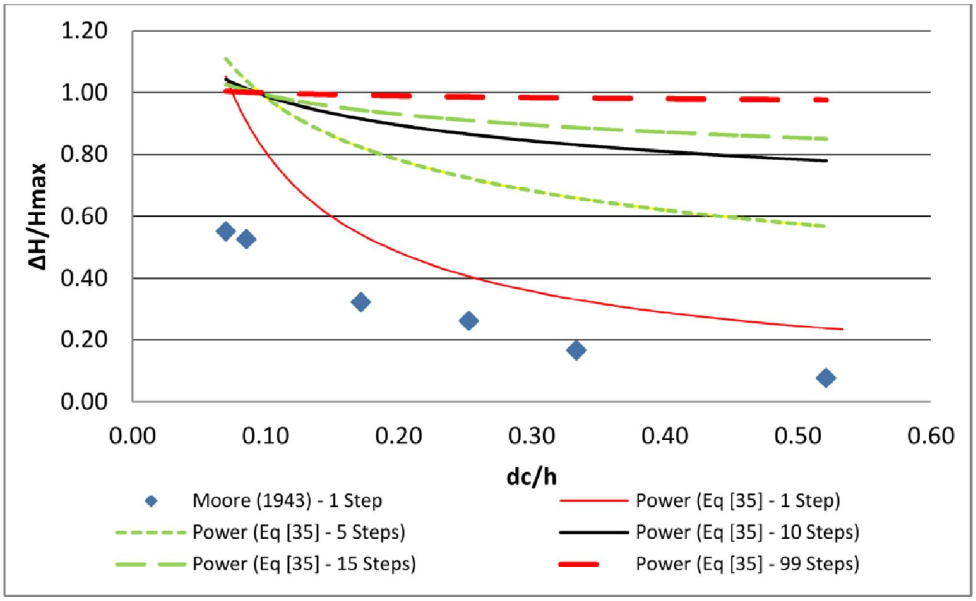

Fig.4. Variation of relative head loss $\Delta H/Hmax$with d_c/h (= 0.07 – 0.52) for e = 0.95, Fr_1 = 6.0, and N between 1 and 15.

Fig 5 depicted energy loss rates against flow rates plotted with the data from Rand (1955) with N = 1 and the data sets from Eq [35] with the parameters of e = 0.70, Fr_1 = 16.0, N = 1. As shown in the chart, the measured data sets and the data sets from Eq [35] with e = 0.70, Fr_1 = 16.0, and N = 1 were in good agreement. These parameters were, therefore, adopted for use in Eq [10] to produce the charts in Figures 5 and 6.

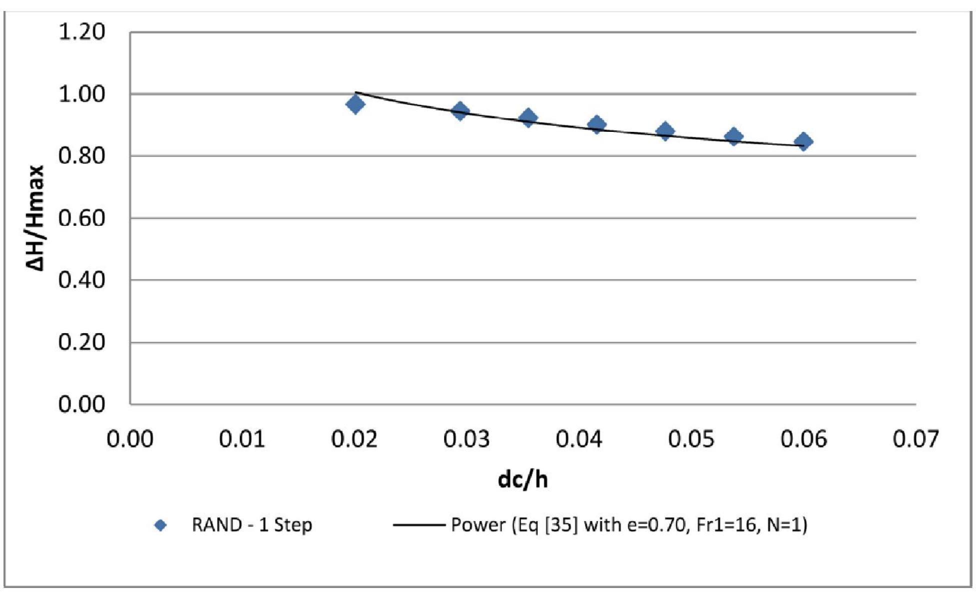

Fig.5. Variation of relative head loss $\Delta H/Hmax$with d_c/h (= 0 .02 – 0.06) for e = 0.70, Fr_1 = 16.0, N = 1.

Fig 6 depicted energy losses rates against flow rates plotted with the data sets from Rand (1955) with N = 1 and the data sets from Eq [10] with e = 0.70, Fr_1 = 16.0 and N = 1, 5, 10, and 15.

The new model predicted rates of energy dissipation predicted rates of energy dissipation that compared well with the measured data for all values of d_c/h between 0.018 and 0.062.

The chart showed that about 100% energy dissipation rates were achieved with number of steps, N = 15. Hence, 15 is the optimal number of steps needed to achieve about 100% rates of energy dissipation with values of d_c/h between 0.018 and 0.062.

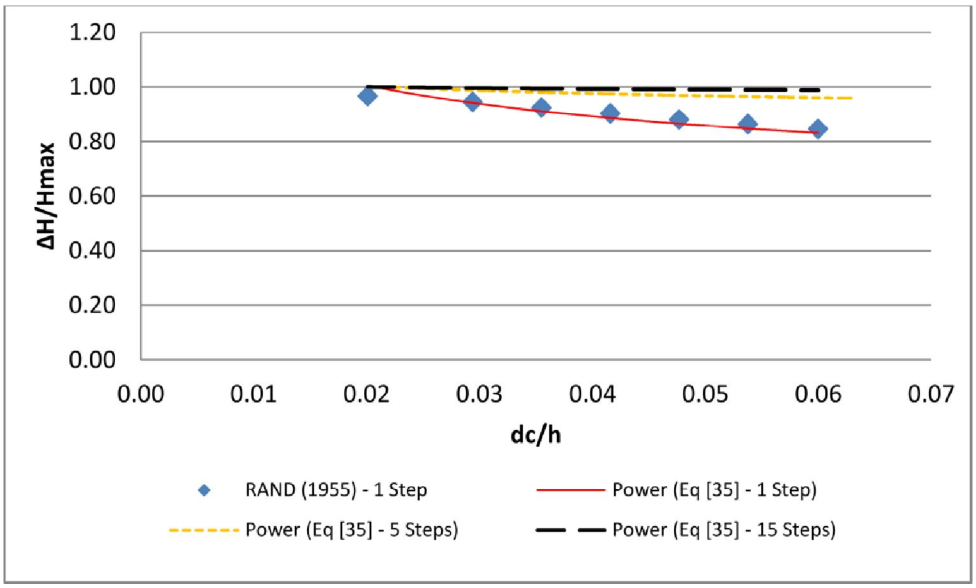

Fig.6. Variation of relative head loss $\Delta H/Hmax$ with d_c/h (= 0.02 – 0.06) for e = 0.70, Fr_1 = 16.0, N between 1 and 15

Fig 7 depicted energy losses rates against flow rates plotted with the data from Stephenson (1979) with N = 1, the data sets from Eq [35] with e = 0.95, Fr_1 = 5.0, N = 1 as well as the data sets from Eq [35] with e = 0.90, Fr_1 = 5.0, N = 1. As shown in the chart, the measured data sets and the data sets from Eq [10] with e = 0.90, Fr_1 = 6.0, and N = 1 were in close agreement. These parameters were, therefore, adopted for use in Eq [35] to produce the charts in Figures 7 and 8.

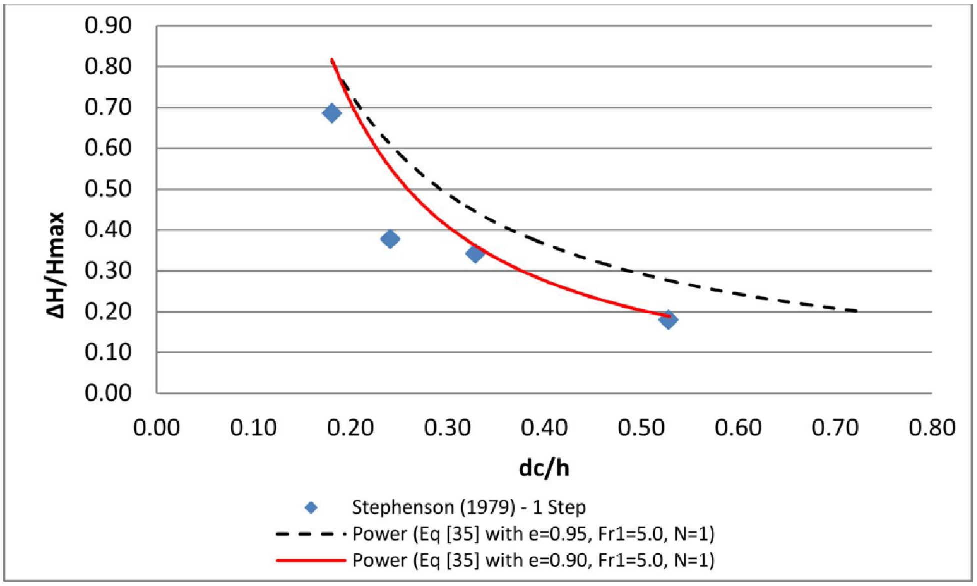

Fig.7. Variation of relative head loss $\Delta H/H_{max}$ with d_c/h(= 0.18 – 0.53) for 0.90 < e ≤ 0.90, Fr_1 = 5.0, N = 1

Fig 8 depicted energy losses rates against flow rates plotted with the data sets from Stephenson (1995) with N = 1 and the data sets from Eq [35] with of e = 0.90, Fr_1 = 5.0 and N = 1, 5, 10, 15, and 20. The new model predicted rates of energy dissipation that agreed with measured data for values of d_c/h of 0.18 and 0.52.

The chart showed that about 100% energy dissipation rates were achieved with about number of steps, N = 99. Hence, 99 is the optimal number of steps needed to achieve about 100% rates of energy dissipation with values of d_c/h between 0.18 and 0.52.

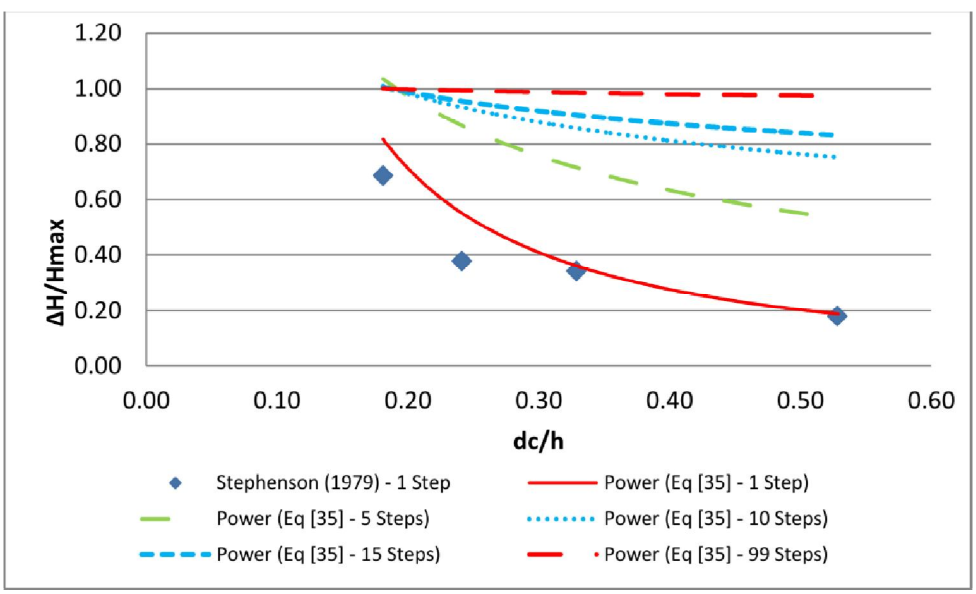

Fig.8. Variation of relative head loss $\Delta H/Hmax$ with d_c/h (= 0.18 – 0.52) for e = 0.90, Fr_1 = 5.0, and N between 15 &99.

Fig 9 depicted energy losses rates against flow rates plotted with the data sets from Moore (1943), Rand (1955), Stephenson (1979), and the data sets from Eq [35] with e = 0.95, Fr_1 = 6.0, N = 1; e = 0.70, Fr_1 = 16.0, N = 1, and e = 0.90, Fr_1 = 5.0, N = 1. Data sets from the new model compared well with measured data sets.

As indicated in the chart, measured data sets lay within the lower and upper boundaries of the derived parameters. Model with parameters e = 0.95, Fr_1 = 5.0, N = 1 and e = 0.90, Fr_1 = 5.0, N = 1 boundedit at the top, while those with e = 0.70, Fr_1 = 16.0, and N = 1bounded it below.

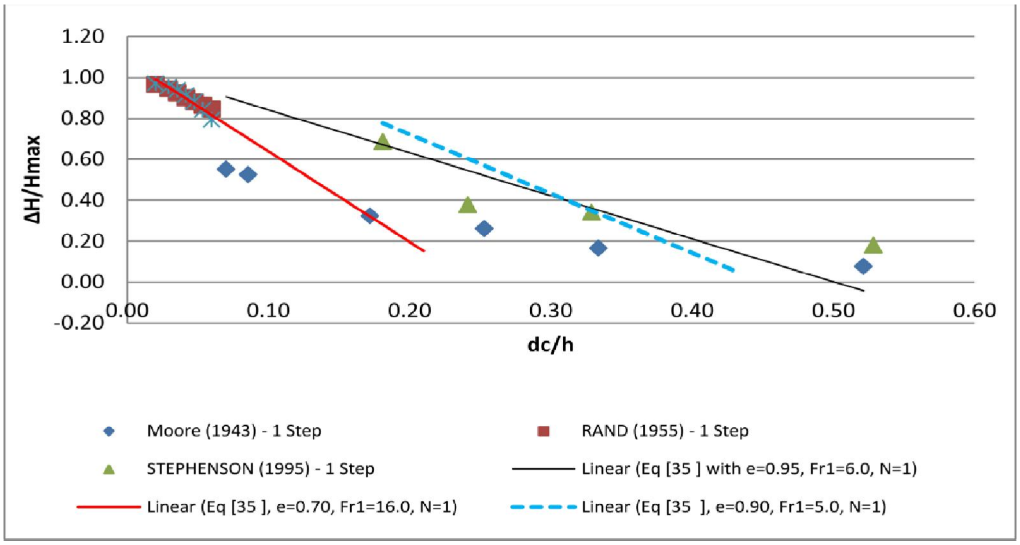

Fig. 9. Variation of relative head loss $\Delta H/Hmax$ with d_c/h(= 0.02 – 0.53) for e = 0.95, Fr_1 = 5.0, the measured data - MOORE (1943), RAND (1955), STEPHENSON (1995).

It is, therefore, recommended as follows:

a) That these parameters, e = 0.95 and Fr_1 = 5.0, be used in Eq [35] when d_c/h is between 0.18 and 0.53 and

b) That these parameters, e = 0.70 and Fr_1 = 16.0, be used in Eq [35] when d_c/h is between 0.02 and 0.06.

4. CONCLUSION

A simple model, Eq [35], is developed for energy dissipation rates in a stepped spillway with a developed hydraulic jump. The rates of energy losses increased with an increasing number of steps and decreased with increasing discharges, which agreed with previous researchers' works. New model data sets generally agreed with measured data sets:The model predicted energy dissipation rates compared well; i) with values of e = 0.95, Fr_1 = 6.0, and N between 1 and 99 for d_c/h between 0.07 and 0.53. The optimal number of steps needed here for 100% energy dissipation is 99; ii) with values of e = 0.70, Fr_1 = 16.0 and N between 1 and 15 for dc/h of 0.018 and 0.063. The optimal number of steps needed here for 100% energy dissipation is 15; and iii) with values of e = 0.90, Fr_1 = 5.0, and N between 1 and 99 for dc/h between 0.18 and 0.2. The optimal number of steps needed for 100% energy dissipation is 99.

ACKNOWLEDGEMENTS

We acknowledge the fruitful discussions with Prof Hubert Chanson of the Department of Civil Engineering, The University of Queensland, Brisbane QLD 4072 and Prof Cross Rodney of the Department of Physics, The University of Sydney, Sydney, NSW 2006, Australia

COMPETING INTERESTS

The authors declare that there are no competing interests.

AUTHORS' CONTRIBUTIONS

Author 1 designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author 2 managed the analyses of the study. All authors read and approved the final manuscript.

REFERENCES

Agunwamba, J (2007), Engineering Mathematical Analysis, pgs 479 – 510, 674 – 675 (ISSN 978-8137-08-3)

André Het. at. (2009), Striking results with bouncing balls, See discussions, stats, and author profiles for this publication at: https://www.researchgate.net/publication/228862688

Boes, R., and Hager, H. (1998).Fiber optical experimentation in two phase cascade flow.In *Proc. Intl. RCC Dams Seminar*. K. Hanson, ed. Denver, Colo.: Schnabel Engineering.

Boes, R. M. (2000). Scale effects in modeling two phase stepped spillway flow. In *Proc. Intl. Workshop on Hydraulics of Stepped Spillways*, 53 - 60. H. E. Minor and W. H. Hager, eds. Steenwijk, the Netherlands: A. A. Balkema.

Berdeni,Y, Champneys, A, and. Szalai. R. (2015), The two-ball bounce problem Department of Engineering Mathematics, University of Bristol, Queen's Building, University Walk, Bristol BS8 1TR, UK, Published by the Royal Society

Chanson, H and Toobes L, ''Flow Patterns in Nappe Flow Regime Down Low Gradient Stepped Chutes''.Journal of Hydraulic Research No 46, No 1 (2008), pp 4 – 44 @ International Association of Hydraulic Engineering and Research.

Felder, S., and Chanson, H. (2008). Turbulence and turbulent length and time scales in skimming flows on a stepped spillway: Dynamic similarity, physical
modeling, and scale effects. Queensland, Australia: University of Queensland, Division of Civil Engineering.

CAROSI, G. and CHANSON, H. (2006)."Air-Water Time and Length Scales in Skimming Flows on a Stepped Spillway.Application to the Spray Characterisation." Report No. CH59/06, Div. of Civil Engineering, The University of Queensland, Brisbane, Australia, July, 142 pages (ISBN 1864998601).

Chanson, H. Hydraulics of Nappe Flow Regime Above Stepped Chutes And Spillways

Chanson, H. (2002).*The Hydraulics of Stepped Chutesand Spillways.*Steenwijk, The Netherlands: A. A. Balkema.

Chanson, H., and Toombes, L. (2002). Energy dissipation and air entrainment in a stepped storm waterway: An experimental study. *J. Irrig. and Drainage Eng. ASCE* 128(5): 305- 315.

Chanson, H.(2002) *The Hydraulics of Stepped Chutes and Spillways.* Lisse, the Netherlands: Balkema.

Chanson, H (2002), Hydraulics of Stepped Spillways: Current Status, *Journal of Hydraulic Engineering,* 126(9), 2000, pp. 636-637.

Chanson, H. (2000). Characteristics of skimming flow over stepped spillways: Discussion. *J. Hydraul. Eng. ASCE* 125(4): 862- 865.

Chanson, H (1997). ''Model Study of a Roller Compacted Concrete Stepped Spillway.'' Journal of Hydraulic Engineering, ASCE, Vol 123, No 10, pp, 931- 933 (ISSN 0733 – 9429).

Chanson, H. (1996). ''Prediction of the Transition Nappe/Skimming Flow On a Stepped Channel' Journal of Hydraulic Research, Vol. 34, 1996 No 3.

Chanson, H (1994), Comparison of energy dissipation between nappe and skimming flow regimes on stepped chutes, *Journal of hydraulic research,*32 (2), 1994, pp. 213–218.

Chanson, H. (1994a). Hydraulics of skimming flows over stepped channels and spillways. *IAHR J. Hydraul. Res.* 32(3): 445- 460.

Chanson, H. (1994b).*Hydraulic Design of Stepped Cascades, Channels, Weirs, and Spillways*. Oxford, U.K.: Pergamon.

Chanson, H (1994), ''Hydraulics of Nappe Flow Regime above Stepped Chutes and Spillways'' Aust, Civil EnggTranports, I, E, Aust, CE 36 (1), 69 -76

CHANSON, H. (1993). "Stepped Spillway Flows and Air Entrainment." *Can. Jl of Civil Eng.*, Vol. 20, No. 3, June, pp. 422-435 (ISSN 0315-1468). JOURNAL DE RECHERCHES HYDRAUUQUES, VOL. 34, 1996, *'Vol.* 3 Prediction of the transition nappe/skimming flow on a stepped channel

Chanson, H (1988), ''A Study of Air Entrainment and Aeration Devices on a Spillway Model''.Ph.D Thesis, Ref 88 – 8, ''Department of Civil Engineering University of Canterbury, New Zealand. Hydraulic of Nappe Flow Regime Above Stepped Chute and Spillways

Chow, V. T. (1959). *Open Channel Hydraulics.* Boston, Mass.: McGrawHill.

Christodoulou, G. C. (1993). Energy dissipation on stepped spillways.*J. Hydraul. Eng. ASCE* 119(5): 644- 655.

Cross R 2019 Rolling over an obstacle Eur. J. Phys.

Cross R and Crawford L (2019), Collision of a ball with the edge of a step,European Journal of Physics

Cross R 2019 Rolling over an obstacle Eur. J. Phys. 40 Cross R (2010), The polar moment of inertia of striking implements Sports Technology 3 215–9 [9] Cross R and Nathan A 2007 Experimental study of the gear effect in ball collisions Am. Jnl Phys. 75

Cross R (2014) Oblique bounce of a rubber ball Exptl. Mech. 54 1523–36

Cross R (2015), Behavior of a bouncing ball, Department, University of Sydney, Sydney NSW 2006, Australia. See discussions, stats, and author profiles for this publication at: https://www.researchgate.net/publication/275467325

FELDER, S. and CHANSON, H. (2012). "Air-Water Flow Measurements in Instationary Free Surface Flows: a Triple Decomposition Technique." Hydraulic Model Report No. CH85/12, School of Civil Engineering, The University of Queensland, Brisbane, Australia

FELDER, S. and CHANSON, H. (2011a). "Air-Water Flow Properties in Step Cavity down a Stepped Chute." International Journal of Multiphase Flow, Vol.

FELDER, S. and CHANSON, H. (2011b)."Energy Dissipation down a Stepped Spillway with Non-Uniform Step Heights." Journal of Hydraulic Engineering, ASCE, Vol. 137, No. 11, pp. 1543- 1548 (DOI 10.1061/(ASCE)HY.1943-7900.0000455).

FELDER, S. and CHANSON, H. (2009b)."Turbulence, Dynamic Similarity and Scale Effects in High-Velocity Free-Surface Flows above a Stepped Chute." Experiments in Fluids, Vol. 47, No. 1, pp. 1-18.

2006 World Environ. and Water Resources Congress, ASCE Conf., CD-ROM. Reston, Va.: ASCE.

MártonGruiz,Tamás Meszéna2 and TamásTél (2017), Chaotic or just complicated? Ball bouncing own the stairs, IOP Publishing, European Journal of Physics, Eur. J. Phys. 38 (2017) 055003 (15pp)

Peyras, L., P. Royet, and Degoutte, G. (1992).Flow and energy dissipation over stepped gabion weirs.*J. Hydraul. Eng. ASCE*118(5): 707-717.

Rice, C. E., and Kadavy, K. C. (1996). Model study of a roller compacted concrete stepped spillway. *J. Hydraul. Eng. ASCE*122(6): 92- 297.

LISTS OF SYMBOLS

e – Coefficient of restitution (COR) assumed constant and less than 1 because of the elastic bounce of ball;
d_c – critical height of ball (m);
d_o - uniform ball depth at the weir (m);
Fr - Froude Number;
h - step height (m), which is also equals to the head loss at any intermediary step ;
V_b - incident ball velocity (m/s^2);
V_c - critical ball velocity (m/s^2)
H_1 – residual head at the bottom of the spillway (m);
ΔH – difference between the maximum head and the residual head (m);
H - total head (m);
H_{max} - maximum head available (m):
$H_{max} = H_{dam} + 3/2 * d_c$;
Hres - residual head at the bottom of the spillway (m);
h - height of steps (m);
l - horizontal length of steps (m);
Q - discharge (m^3/s);
q - discharge per unit width (m^2/s);
Reynolds number defined as : $Re = \rho_w * U_w * D_H / \mu_w$
U_w - flow velocity (m/s): $U_w = q_w/d$;
W - channel width (m);
ΔH - head loss (m);
μ - dynamic viscosity ($N.s/m^2$);
ρ - density (kg/m^3);

Subscript
a – conditions at ball bounce;
b – conditions at step brink;
c – conditions at critical height;
d – conditions at maximum ball height;
N– number of step;