Original Research Article

Determination of energy dissipation in a nappe flow with a fully developed hydraulic jump on a stepped spillway using an elastic ball that bounces down a rigid stair.

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ABSTRACT

In this paper, the authors reviewed the characteristics of an elastic pinball that descended one step each bounceof a flight of stair and used it to draw an analogy with the nappe flow regime with a fully developed hydraulic jump down a stepped spillway.They used this concept with the equations of Motion, Classical Hydraulic Jump, and the Chanson's Model for ~~rates of energy dissipation in nappe flow regime to formulate a model for the rates of energy dissipation~~energy dissipation rates in nappe flow regime to formulate a model for the energy dissipation rates for the nappe flow regime.They also used over 30 measured data obtainedfrom the experimental works of Moore (1943), Rand (1955), and Stephenson (1995) on the rates of energy dissipation on drop structures to obtain the critical parameters of this model, which is simple and easy to use. These parameters, which included the Coefficients of Restitution (e) and the approach Froude Numbers (Fr_1) were found to range between 0.70 and 0.95 for 'e' and 5.0 and 16.0 for Fr_1. This model, which wasthen validated with the experimental data on drop structures from Moore (1943); Rand (1955); Stephenson (1979)and found to be in good agreement with them, was later recommended for use on stepped spillways with 1 step or more.This developed model was later calibrated and yielded very high coefficients of correlation that ranged from 0.88 to 0.92, which upon verification, gave good predictions between the measured and estimated data. Results showed that the rate of energy dissipation along a stepped spillway increases with increasing numbers of steps~~,~~ but decreases with increasing discharges.

Keywords: Energy Dissipation, Stepped spillway, Nappe flow, jet length, minimum chute length, hydraulic jump length

Introduction

In this paper, the authors reviewed the characteristics of nappe flow with a fully developed hydraulic jump on the stepped chute and drew an analogy with an elastic pinball that descended one step each bounce, and used the concept to develop a model for the rates of energy dissipation for nappe flow with a hydraulic jump.

During the 19th century, overflow stepped spillways were selected frequently with nearly one-third of dams built in Europe and North America equipped with a stepped cascade. More recently, the 1980s and 1990s were marked by a regain of interest for that type of spillway design (Chanson, 2001). Most structures were steep chutes for gravity dams operating in a skimming flow regime. For relatively low flow rates, however, the waters cascade down a stepped chute as a succession of free-falling nappes: i.e., nappe flow regime. Researchers such as Chanson (1994) have classified flows through Stepped spillways into three regimes namely a) Nappe flow regime with a fully developed hydraulic jump, b) Nappe flow regime with a partial flow regime and, c) skimming flow regime. The nappe flow regime is distinguished by a series of plunges from one step to another with the formation of a nappe at each drop. This type of flow can be approximated by a series of single-step drop structures (Chamani and Rajaratnam, 1994; Chanson, 1993). The flow leaves the step as a free-falling jet and impinges on the tread of the next step. Energy dissipation occurs by the jet breakup, jet mixing on the step, and the formation of a partially or fully developed hydraulic jump on the step (Chanson 1994; Rajaratnam 1990). In the case of a fully developed hydraulic jump (Image 1), referred to as isolated nappe flow (Essery and Homer 1978; Peyras et~~.~~ al., 1992), the flow passes through critical depth at the brink of the step forming a supercritical free-falling jet and returns to subcritical flow downstream of the jump. For a horizontal uniform step, the flow depth at the brink of the $d_b = 0.715 * d_c$, where d_c is the critical flow depth (Rouse, 1936).

Similarly, the authors assumed that the ball bouncing down from each step hit the step below as a ~~free-~~free-falling jet, with energy dissipation occurring by the collision of the elastic ball with the horizontal step surface and ~~bounced~~ bouncingoff and descended on the next lower step.

In a nappe flow situation with a fully developed hydraulic jump, the head loss at any intermediary step is equal to the step height. Hence, the total head loss, ΔH, along the spillway is equal to the difference between the maximum head available, $H_{max,}$ and the residual head, H_e, at the bottom of the spillway $H_{1.}$ (Chanson,1994).

The authors studied the simplest model of a ball bounce down the stairs by considering an elastic pinball; the bounces~~, which~~ were assumed to occur elastically with some energy loss. For simplicity, the authors assumed that the COR was constant during the entire bounce dynamics and took on a positive value 'e' less than unity. They considered a stairway, which consisted of horizontal and vertical parts only, and defined an unbounded billiard problem in the presence of gravity. As in billiards, no friction is assumed between the ball or the stairway, and rotation can be neglected. Collisional energy loss that represented the effect of dissipative processes within the ball via the COR, is natural and can compensate for the increase in kinetic energy as the height decreases. Thus, a steady-state might set in, supporting an ever-lasting translational motion.

Although many researchers have investigated the hydraulic performance of stepped spillways, significant information gaps still exist~~ed~~ in the guidelines for the design of stepped spillways. Hence, the development of this new, simple, and ~~ease~~ easyto use model, which the authors believed would provide quick analytical information to engineers involved in the design of spillways.

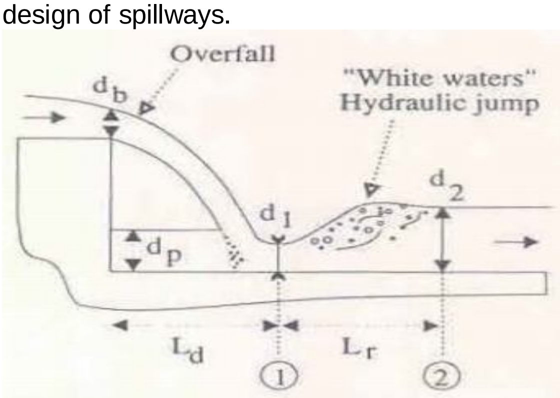

Image. 1. Nappe flow regime (Flow at a drop structure)

2. METHODOLOGY

The authors used the Newton Law of Motion, the classical hydraulic jump equation (Eq [5]), and the energy dissipation at nappe flow equation (Eq [9]) to develop this model. They used the law of motion to formulate the trajectory of an elastic pinball that bounced down a drop structure of horizontal step, l, and height, h, (Fig 1) and later extended the concept to flight of stairs made up of multiples of similar drop structures.

The authors assumed: that the pinball descended one step each bounce, d_1, and that after each bounce, it rebounded to a height, d_2, above the next lower step; that the height, d_2,was large enough compared with the width of the step so that the impacts were effectively head-on; that the coefficient of restitution, e, is constant and is less than 1; that ~~the~~ the trajectory of a freely falling elastic ball that descended one step each bounce would be similar to the trajectory of a nappe flow regime which is distinguished by a series of plunges from one step to another with the formation of a nappe at each drop; that the pinball also passed through a critical depth at the brink of each step forming a supercritical free-falling drop and returned to subcritical flow downstream of the jump; and that the way energy is dissipated in a nappe flow regime with a full hydraulic jump would be similar to the way energy was lost when a freely falling pinball descended one step each bounce.

The origin of the coordinates was maintained at the crest of the stair, with the coordinates of the n^{th} and n^{th+1} collision shown as x_n and x_{n+1}, respectively and their corresponding vertical velocities after bounce were shown as v_n and v_{n+1}. Since the coefficient of restitution, e, was assumed constant and less than 1, because of the elastic bound of the ball, the vertical component of the rebound velocity 'v' of the incident velocity of the ball became 1 < times smaller during each bounce, while the horizontal component u > 0 remained constant. The origin of both the x and y coordinates ~~were~~ wasfixed at the top of the weir for all the ball bounces.

Since the collision occured with ~~the~~ the vertical component of the incidence velocity, the rebound vertical velocity became 'e' times the opposite of this bound vertical velocity, while the horizontal distance the pinball travelled from d_1 to d_2 or x, became 'ut', measured from the edge of the step where bounce occured.

The ball bounced off the step tread through the height,

$$d_2 - d_1 \quad [1]$$

And landed on the next lower step through the height,

$$d_b + h - d_1 \quad [2]$$

where d_b, h, d_1, and d_2, respectively are the flow depth at the brink of the horizontal step, the height of step, the depth of bounce at section 1, and the depth of bounce at section 2 (Figure 1). The coefficient of restitution denoted by 'e' - and ranges between 0 and 1 - is the ratio of the final to initial relative velocity between two objects at collusion is defined as

$$e = \sqrt{\frac{(d_2 - d_1)}{(d_b + h - d_1)}} \qquad [3]$$

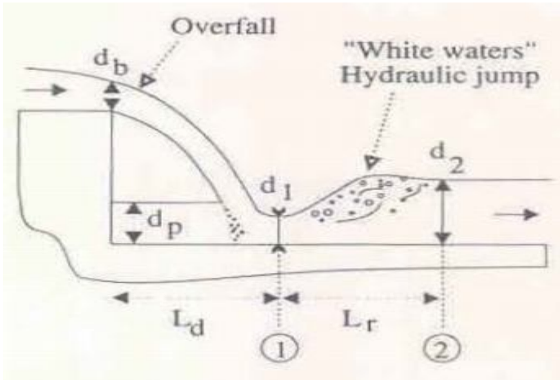

Fig 1. Flow at a drop structure

2.1 Formulation of model

Simplifying, Eq [3] yields

$$d_1 = \frac{(d_2 - e^2 d_b - e^2 h)}{(1 - e^2)} \qquad [4]$$

The flow depth, $d_{1,}$ at Section 1 and the flow depth, d_2, ast Section 2 (Fig 1) are related by the classical hydraulic jump equation

$$\frac{d_2}{d_1} = \frac{1}{2}\left\{\sqrt{1 + 8Fr_1^2} - 1\right\} \qquad [5]$$

The flow depth, d_b, at the brink of the horizontal step is given by ROUSE (1936) as

$$d_b = 0.715 d_c \qquad [6]$$

where d_c is the critical flow depth.

Substituting *equa*tions [5] and [6] into equation [4] and simplifying yields equation [7].

$$d_1 = \frac{\left(\frac{d_1}{2}\left\{\sqrt{1 + 8Fr_1^2} - 1\right\} - e^2 0.75 d_c - e^2 h\right)}{(1 - e^2)(e^2 - 1)}$$

$$d_1 = \frac{-e^2(0.715 d_c + h)}{(1 - e^2)(e^2 - 1) - 0.5\left(\sqrt{1 + 8Fr_1^2} - 1\right)} \qquad [7]$$

Dividing Eq [7] by d_c, gives the dimensionless Eq [8]

$$\frac{d_1}{d_c} = \frac{-e^2\left({0.715 d_c}/{d_c} + {h}/{d_c}\right)}{(1 - e^2) - 0.5\left(\sqrt{1 + 8Fr_1^2} - 1\right)} \qquad [8]$$

and finally substituting Eq [8] in Eq [9], yields Eq [10].

In a nappe flow regime with a fully developed hydraulic jump, the head loss at any intermediate step is equal to the energy loss. The total head loss, ΔH, along the spillway is equal to the maximum head available, H_{max}, and the residual head, H_1, at the bottom of the spillway (Chanson, 1994).

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\frac{d_1}{d_c} + \frac{1}{2}\left(\frac{d_c}{d_1}\right)^2}{\frac{3}{2} + \frac{H_{dam}}{dc}} \qquad [9]$$

where H_{dam} is the dam height. For an un-gated spillway, the maximum head available and the dam height are related by: $H_{dam} + 1.5d_c$. The residual energy is dissipated at the toe of the spillway by a hydraulic jump in the dissipation basin (Chanson, 1994).

$$\frac{\Delta H}{H_{max}} = 1 - \frac{A + B}{\frac{3}{2} + \frac{Nh}{d_c}} \qquad [10]$$

where

$$A = \frac{-e^2\left(0.715 + {}^{h}/_{d_c}\right)}{(1 - e^2) - 0.5\left(\sqrt{1 + 8Fr_1^2} - 1\right)}$$

$$B = \frac{1}{2}\left(\frac{(1 - e^2) - 0.5\left(\sqrt{1 + 8Fr_1^2} - 1\right)}{e^2\left(0.715 + {}^{h}/_{d_c}\right)}\right)^2$$

and N = the number of step.

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3. RESULTS AND DISCUSSION

3.1: Determination of the appropriate values of the Coefficients of Restitution (e) and the Froude No (Fr_1)

a) Using Eq [10] and the measured data from Moore (1943), the values of e = 0.95 and Fr_1 = 6.0 for the best fit were selected from Fig 2. These values were then substitutedin Eq [10] to produce to Fig. 3,

b) Using Eq [10] and the measured data from Rand (1955), the values of e = 0.70 and Fr_1 = 16.0 for the best fit were selected from Fig 4. These values were then substituted in Eq [10] to produce to Fig. 5,

c) Using Eq [10] and the measured data from Stephenson (1979), the values of e = 0.90 and Fr_1 = 5.0 for the best fit were selected from Fig 6. These values were then substituted in Eq [10] to produce to Fig. 7.

Fig 2 depictedenergy losses ratesagainst flow rates plotted with the data from Moore (1943), the data sets fromEq [10] with the parameters of e = 0.95, Fr_1 = 5.0, N = 1 as well as the data sets from Eq [10] with the parameters of e = 0.95, Fr_1 = 6.0, N = 1. Thecurves showed some traditional concave shape distributions for all the two plotted data which were in line with earlier studies (Chanson, 2001). As shown in the chart, the measured data sets and the data sets from Eq [10] with the parameters of e = 0.95, Fr_1 = 6.0, and N = 1, were in closeagreement. These parameters were, therefore, adopted forused in Eq [10] to produce the charts of Figures 2 and 8.

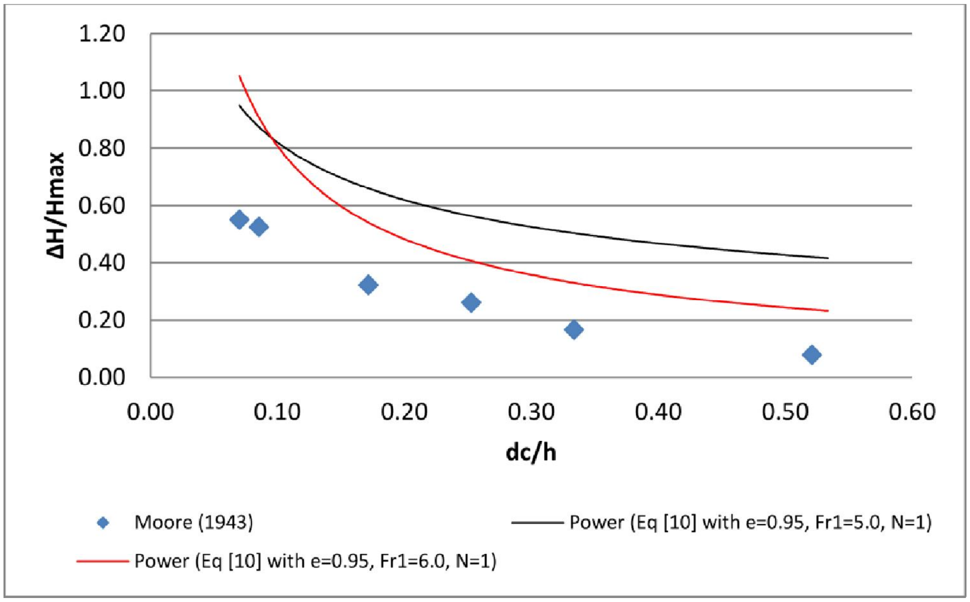

Fig 2. Variation of relative head loss, $\Delta H/Hmax$,with d_c/h(= 0.07 – 0.52) for e = 0.95, 5.0 < $Fr_1 \leq$ 6.0, N = 1

Fig 3 depicted energy losses rates against flow rates plotted with the data sets from Moore (1943) and the data setsfrom Eq [10] with the parameters of e = 0.95, Fr_1 = 6.0 and N = 1, 5, 10, 15, and 20.The figure showed similar traditional concave shape distributions for all the four plotted data for energy dissipation for all the flow rates (Chanson, 2001). As indicated in the chart, energy losses increased with increasing number of steps for a particular discharge which is in accordance with, per the earlier investigations (Matos, 2000; Chanson, 2001b; Felder & Chanson, 2009a).

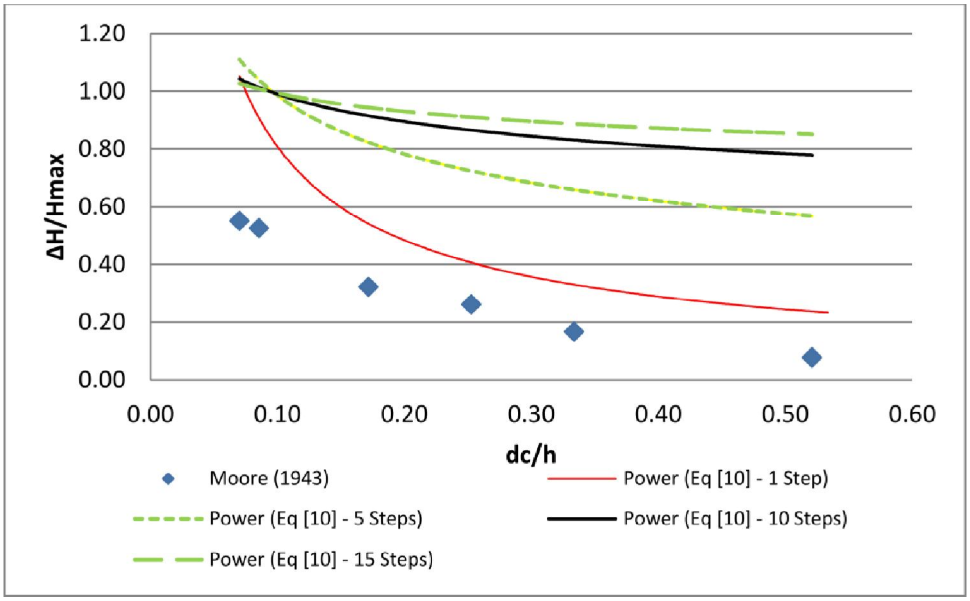

Fig 3. Variation of relative head loss $\Delta H/Hmax$with d_c/h (= 0.07 – 0.52) for e = 0.95, Fr_1 = 6.0, and N between 1 and 15.

Fig 4 depicted energy losses rates against flow rates plotted with the data from Rand (1955) and the data sets from Eq [10] with the parameters of e = 0.70, Fr_1 = 16.0, N = 1. The curve showed some traditional concave shape distribution for the plotted data which were in line with earlier studies (Chanson, 2001). As shown in the chart, the measured data sets and the data sets from Eq [10] with the parameters of e = 0.70, Fr_1 = 16.0, and N = 1, were in good agreement. These parameters were, therefore, adopted for used in Eq [10] to produce the charts of Figures 5 and 8.

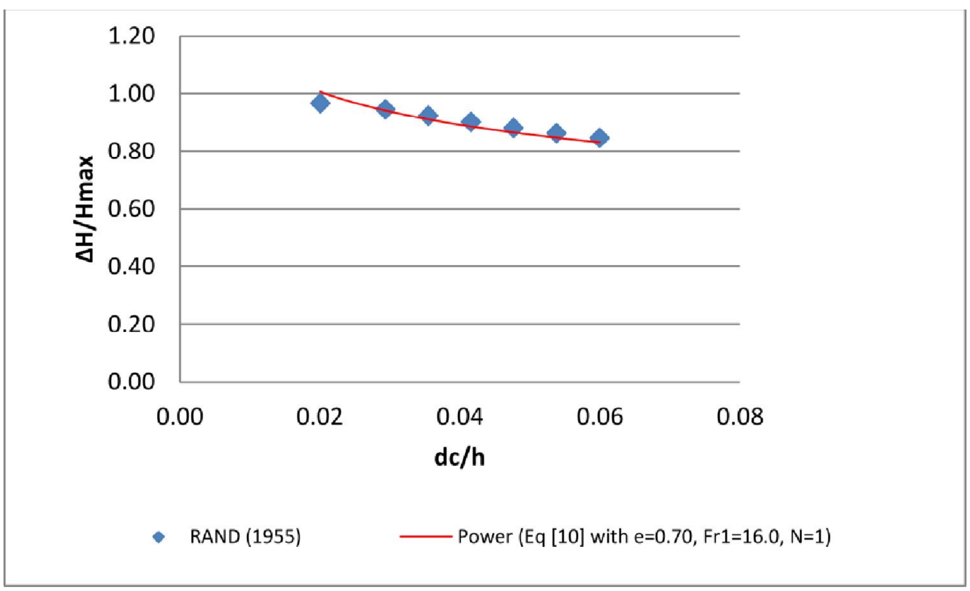

Fig 4. Variation of relative head loss $\Delta H/Hmax$with d_c/h (= 0 .02 – 0.06) for e = 0.70, Fr_1 = 16.0, N = 1.

Fig 5 depicted energy losses rates against flow rates plotted with the data sets from Rand (1955) and the data sets from Eq [10] with the parameters of e = 0.70, Fr_1 = 16.0 and N = 1, 5, 10, and 15. The figure showed similar traditional concave shape distributions for all the three plotted data for energy dissipation for all the flow rates (Chanson, 2001). As indicated in the chart, energy losses increased with increasing number of steps for a particular discharge ~~which is in accordance with~~, per the earlier investigations (Matos, 2000; Chanson, 2001b; Felder & Chanson, 2009a).

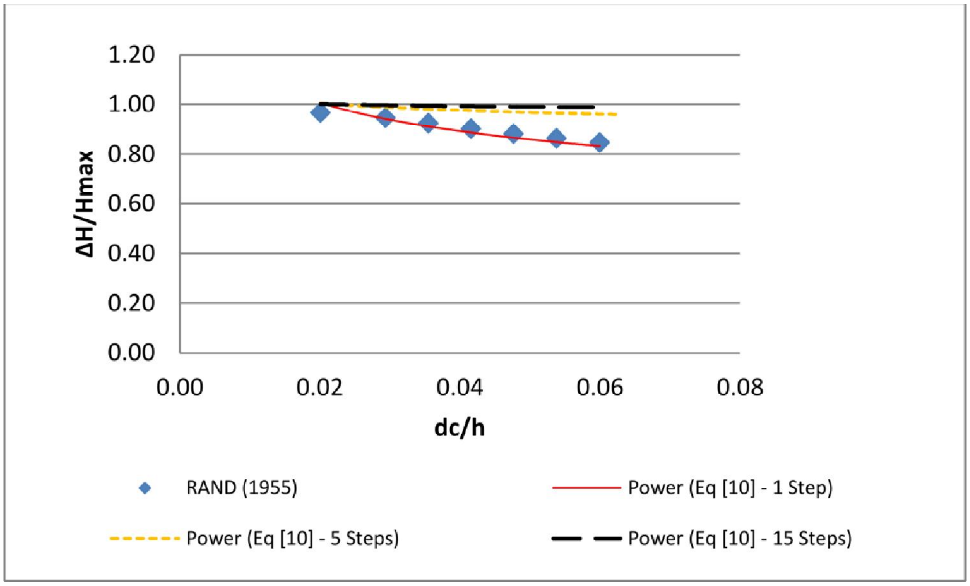

Fig 5. Variation of relative head loss $\Delta H/Hmax$with d_c/h (= 0.02 – 0.06) for e = 0.70, Fr_1 = 16.0, N between 1 and 15

Fig 6 depicted energy losses rates against flow rates plotted with the data from Stephenson (1979), the data sets from Eq [10] with the parameters of e = 0.95, Fr_1 = 5.0, N = 1 as well as the data sets from Eq [10] with the parameters of e = 0.90, Fr_1 = 5.0, N = 1. The curves showed some traditional concave shape distributions for all the two plotted data which were in line with earlier studies (Chanson, 2001). As shown in the chart, the measured data sets and the data sets from Eq [10] with the parameters of e = 0.90, Fr_1 = 6.0, and N = 1, were in close agreement. These parameters were, therefore, adopted for used in Eq [10] to produce the charts of Figures 7 and 8.

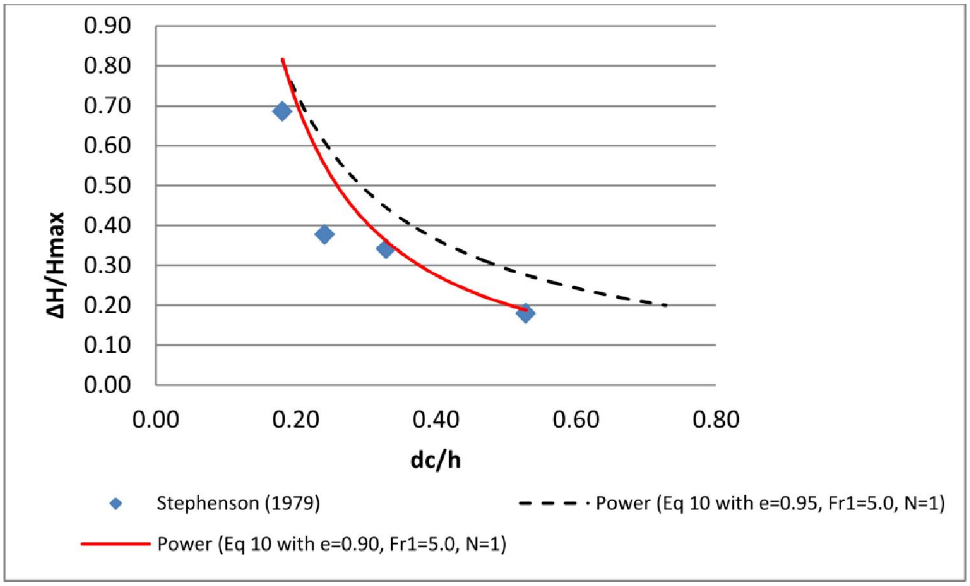

Fig 6. Variation of relative head loss $\Delta H/H_{max}$ with d_c/h(= 0.18 – 0.53) for 0.90 $<$ e $\leq$ 0.95, Fr_1 = 5.0, N = 1

Fig 7 depicted energy losses rates against flow rates plotted with the data sets from Stephenson (1995) and the data sets from Eq [10] with the parameters of e = 0.90, Fr_1 = 5.0 and N = 1, 5, 10, 15, and 20. The figure showed similar traditional concave shape distributions for all the five plotted data for energy dissipation for all the flow rates (Chanson, 2001). As indicated in the chart, energy losses increased with increasing number of steps for a particular discharge ~~which is in accordance with~~, per the earlier investigations (Matos, 2000; Chanson, 2001b; Felder & Chanson, 2009a).

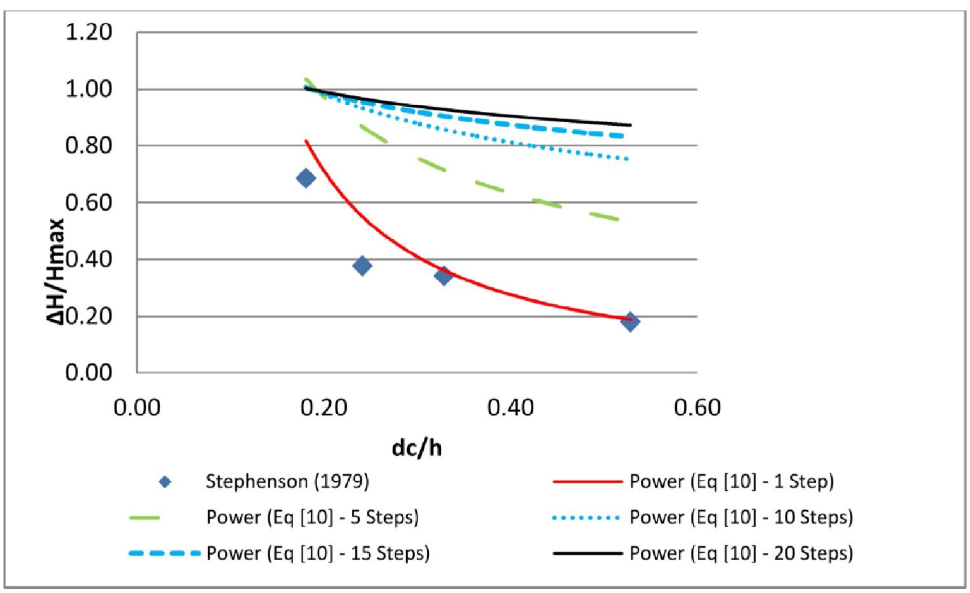

Fig 7. Variation of relative head loss $\Delta \mathrm{H}/\mathrm{Hmax}$with d_c/h (= 0.18 – 0.52) for e = 0.90, Fr_1 = 5.0, N between 15 & 20

Fig 8 depicted energy losses rates against flow rates plotted with the data sets from Moore (1943), Rand (1955), Stephenson (1979) and the data sets from Eq [10] with the parameters of e = 0.95, Fr_1 = 5.0 and N = 1. The figure showed similar traditional concave shape distributions for the plotted data for energy dissipation for all the flow rates (Chanson, 2001). As indicated in the chart, the data sets from Eq [10] with the parameters of e = 0.95, Fr_1 = 5.0 and N = 1 were within the lower and higher boundaries of these measured data sets.

It is, therefore, recommended as follows

a) That these parameters, e = 0.95 and Fr_1 = 5.0, be used in Eq [10] when d_c/h is between 0.18 and 0.53,
b) That these parameters, e = 0.70 and Fr_1 = 16.0, be used in Eq [10] when d_c/h is between 0.02 and 0.06.

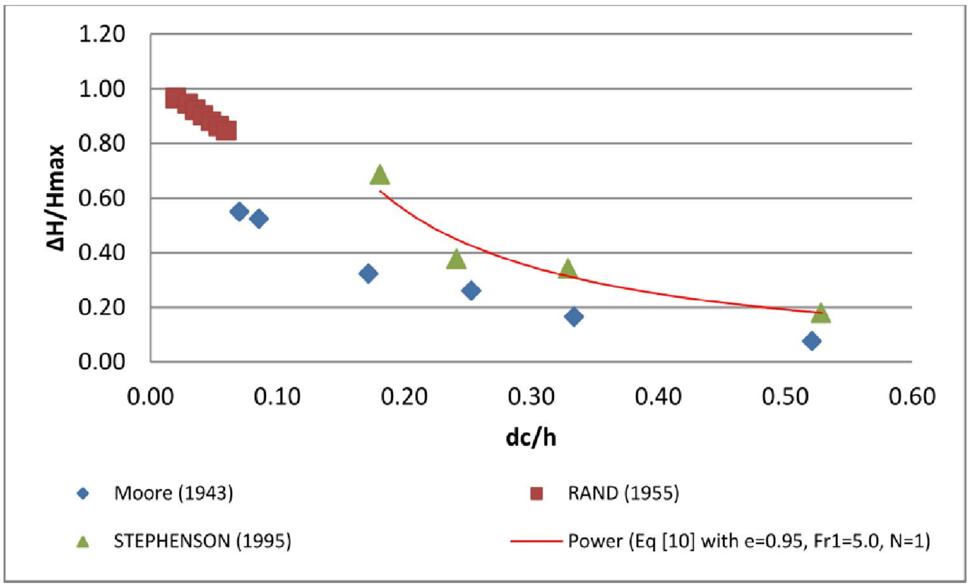

Fig. 8. Variation of relative head loss $\Delta \mathrm{H}/\mathrm{Hmax}$with d_c/h(= 0.02 – 0.53) for e = 0.95, Fr_1 = 5.0, the measured data - MOORE (1943), RAND (1955), STEPHENSON (1995).

4. CONCLUSION

The present studyreviews the characteristics of an elastic pinball that descends one step each bounceof a flight of stair and uses it to draw an analogy with the nappe flow regime with a fully developed hydraulic jump down a stepped spillway. A simple model, Eq [10], is developedto define the ~~rate of energy dissipation~~energy dissipation rate in nappe flow zone. The measured data sets are generally in agreement with the data sets from the developed model (Eq [10]), but further work is needed to comprehend the difference between model and prototype data.As shown in all the charts, energy losses increase with increasing number of steps for a particular discharge, but decrease with increasing discharges.

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LISTS OF SYMBOLS

e – Coefficient of restitution (COR) assumed constant and less than 1 because of the elastic bounce of ball;
d_c – critical height of ball (m);
d_o - uniform ball depth at the weir (m);
Fr - Froude Number;
h - step height (m), which is also equals to the head loss at any intermediary step ;
V_b - incident ball velocity (m/s^2);
V_c - critical ball velocity (m/s^2)
x, y – x, and y axes coordinates of ball for all bounces with origin fixed at the weir(m);
H_1 – residual head at the bottom of the spillway (m);
ΔH – difference between the maximum head and the residual head (m);
H - total head (m);
H_{max} - maximum head available (m):
$H_{max} = H_{dam} + 3/2 * d_c$;
Hres - residual head at the bottom of the spillway (m);
h - height of steps (m);
l - horizontal length of steps (m);
Q - discharge (m^3/s);
q - discharge per unit width (m^2/s);
Reynolds number defined as : $Re = \rho_w * U_w * D_H / \mu_w$
U_w - flow velocity (m/s): $U_w = q_w/d$;

W - channel width (m);
ΔH - head loss (m);
μ - dynamic viscosity ($N.s/m^2$);
ρ - density (kg/m^3);

Subscript
a – conditions at ball bounce;
b – conditions at step brink;
c – conditions at critical height;
d – conditions at maximum ball height;
N– number of step;