Modeling Temporal Variation of Particulate Matter Concentration at Different Locations of Delhi-NCR

ABSTRACT

Key words: ARIMA, GARCH, particulate matter, pollution, time series, volatility

1. INTRODUCTION

The greatest environmental health concern in the modern world is exposure to polluted air [1]. Particulate matters, or PMs for short, are the air pollutants that have the worst effects on human health. Particulate matter is a term used to describe small solids or liquid droplets that enter the human body through the air we breathe and may have either anthropogenic or natural origins. Particles with a diameter of 2.5 to 10 μm (PM_{10}) can penetrate deep inside the lung, whereas $PM_{2.5}$ particles can enter the circulatory system after breaching the lung barrier. One of the most current estimates on mortality brought on by PMs suggests that, globally, fossil fuel-generated $PM_{2.5}$ is responsible for about 8.7 million premature deaths [2]. India is a major contributor as well as victim of air pollution as it is the third largest emitter of greenhouse gases [3] and the fifth largest emitter of $PM_{2.5}$ [4]. In reality, according to the World Health Organization's Air Quality Guidelines, none of the Indian cities have achieved the standard for annual $PM_{2.5}$ concentration of 5 μgm^{-3} [5]. Air pollution is India's second-leading cause of mortality and morbidity, after malnutrition [6]. In India, 17.8% of deaths in 2019 were attributed to air pollution, with outdoor particulate matters responsible for 58.7% of those deaths [7].

Out of the fifteen most polluted cities in the world with respect to annual average $PM_{2.5}$ level, ten are situated in India and eight of them are located in the Delhi-National Capital Region (NCR) [8]. According to the 2021 World Air Quality Report, New Delhi is the fourth most polluted city in the world and the most polluted capital city. It has been well documented that every year since last few decades the average annual $PM_{2.5}$ concentration in Delhi crosses the annual National Ambient Air Quality Standard of 40 μgm^{-3} (which is eight times higher than the WHO's standard limit) [1]. However, despite the adoption of a number of interventions by the government agencies, none of the actions proved to be effective for Delhi-NCR.

Prediction of future particulate matter levels play a crucial role in policy formulation of any country, especially those falling under low- and medium-income group. The present work explores the variation in the concentration of PM_{10} and $PM_{2.5}$ at different sites of Delhi having different pollution signatures. A proper understanding about the concentration of PM_{10} and $PM_{2.5}$ and their variation over different times of a year can be helpful for the

policymakers to take proactive actions to minimize the hazard. Time series modeling approaches are used for the concentration variation study. A time series is the collection of realizations of any variable over a period of time. The most important characteristic of a time series is that the successive realizations are dependent. The time series analysis is pioneered by Box and Jenkins' [9] Autoregressive Integrated Moving Average (ARIMA) methodology. Many times it is seen that the realizations of a time series exhibit long term dependencies. This phenomenon is known as the long memory property. If any time series exhibits the long memory property in its mean structure then instead of ARIMA model, Autoregressive Fractionally Integrated Moving Average (ARFIMA) model [10] is used. The potential presence of long memory in a time series can be tested using [11] GPH statistic. The ARIMA/ARFIMA is a linear model. To address the non-linearity of a time series the Autoregressive Conditional Heteroscedastic (ARCH) [12] and Generalized ARCH (GARCH) [13] models can be used as variance model along with the ARIMA/ARFIMA model as mean model.

2. MATERIAL AND METHODS

2.1. ARFIMA model

An ARIMA model is represented as ARIMA (p, d, q) where p, d and q represents the order of autoregression, integration (differencing), and moving average respectively. For a linear univariate time series process $\{y_t\}$, the ARIMA process is represented as

$$\varphi(L)(1-L)^d y_t = \theta(L)\varepsilon_t \quad (1)$$

where, y_t is the actual observation and ε_t is the error term observed at time t such that $\varepsilon_t \sim IID(0, \sigma^2)$. $\varphi(L)$ and $\theta(L)$ are the polynomial of lag operator L of order p and q respectively. In ARIMA methodology, the order of differencing d is considered as an integer. The only difference of the ARFIMA model is that here d has a fractional value. For the ARFIMA models, the fractional parameter d lies between -0.5 and 0.5 [14].

2.2. The ARCH and GARCH model

A process $\{\varepsilon_t\}$ is said to have an ARCH (q) model if the conditional distribution of $\{\varepsilon_t\}$ given the available information (ψ_{t-1}) up to $t-1$ time epoch can be represented as:

$$\varepsilon_t|\psi_{t-1} \sim N(0, h_t) \; and \; \varepsilon_t = \sqrt{h_t}\nu_t \quad (2)$$

where, ν_t is known as innovation and it is independently and identically distributed (IID) with zero mean and unit variance. The distribution of innovation is data specific.

The conditional variance h_t for an ARCH (q) is represented as

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2, \alpha_0 > 0, \alpha_i \geq 0 \; \forall \; i \; and \; \sum_{i=1}^{q} \alpha_i < 1 \quad (3)$$

For satisfactory model precision, a large number of parameters are needed for an ARCH model. The GARCH model overcome this problem.

The conditional variance of a GARCH (*p,q*) model is defined as

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$$

provided $\alpha_0 > 0, \alpha_i \geq 0 \; \forall \; i, \beta_j \geq 0 \; \forall \; j$ (4)

The GARCH (*p,q*) process is said to be weakly stationary if and only if

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \quad (5)$$

2.3. Data description

Data for $PM_{2.5}$ and PM_{10} concentration (μg m^{-3}) was collected at 24 hours interval from Central Pollution Control Board (CPCB) website (https://app.cpcbccr.com/ccr/#/caaqm-dashboard-all/caaqm-landing/caaqm-comparison-data) for three Delhi Pollution Control Committee (DPCC)-regulated air quality monitoring stations situated at Narela, Okhla and Pusa Road. The collected data spanned over the period of 01 July 2018 to 31 July 2022 (1492 datapoints). The whole data set is divided into two sets namely model building set and model validation set. The last ten observations are used for the validation set and the remaining portion as the model building set.

2.4. Validation

The efficacy of model is tested using three error functions namely, Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE). These error functions are calculated as

$$RMSE = \left[\frac{1}{T}\sum_{t=1}^{T}(y_t - \hat{y}_t)^2\right]^{1/2} \quad (6)$$

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|y_t - \hat{y}_t| \quad (7)$$

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\frac{|y_t - \hat{y}_t|}{y_t} \times 100 \quad (8)$$

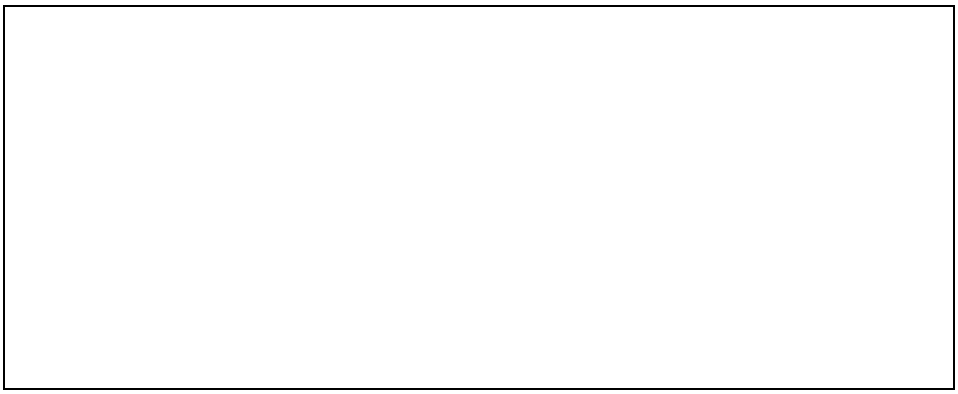
where, y_t is the actual value, $\hat{y}_t$ is the predicted value and T is the horizon of forecast.

3. RESULTS AND DISCUSSION

The descriptive statistics of $PM_{2.5}$ and PM_{10} concentration for the selected location is given in Table 1. It is seen that the mean, median and minimum concentration for both the pollutants follow Narela > Okhla > Pusa. The maximum concentration of $PM_{2.5}$ follows the same order. But the maximum concentration of PM_{10} follows a different order. The PM_{10} concentration has relatively higher S.D. and lower C.V., skewness and kurtosis than the concentration of $PM_{2.5}$ for all three locations. Even kurtosis of PM_{10} at Narela is slightly negative. The time plot of $PM_{2.5}$ and PM_{10} concentration is given in Figure 1. For all the three locations, the PM concentration remained comparatively high between late-October to early-January and November was the month with highest average PM concentration. Both $PM_{2.5}$ and PM_{10} showed comparatively low concentration between early-July to Mid-September. August was the safest month with respect to PM pollution.

Table 1. Descriptive statistics of $PM_{2.5}$ and PM_{10} concentration for the selected locations

Statistics	Narela		Okhla		Pusa	
	$PM_{2.5}$	PM_{10}	$PM_{2.5}$	PM_{10}	$PM_{2.5}$	PM_{10}
Observations	1492	1492	1492	1492	1492	1492
Mean	112.56	234.64	101.55	214.95	95.07	203.05
Median	85.35	209.33	68.06	191.86	66.20	188.77
Minimum	6.62	20.48	6.29	15.52	3.44	9.97
Maximum	689.1	717.35	601.80	884.8	570.61	726.86
S.D.	87.63	131.40	90.00	128.90	81.33	120.55
C.V. (%)	77.85	56.00	88.62	59.97	85.54	59.37
Skewness	1.55	0.72	1.75	0.93	1.68	0.73
Kurtosis	3.02	-0.04	3.41	0.8	3.29	0.34



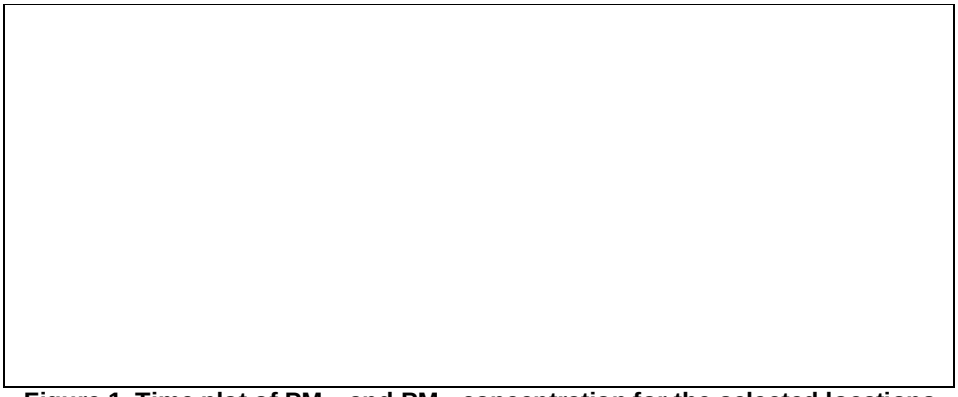


Figure 1. Time plot of $PM_{2.5}$ and PM_{10} concentration for the selected locations

The Shapiro-Wilk test [15] is used to determine if the selected series are normally distributed. The series' normal distribution serves as the test's null hypothesis. It is seen that (Table 2) none of the selected series follow normal distribution at 1% level of significance. The generalized error distribution (GED) is a robust class of distribution. It is considered that all the series follow GED and also the innovations.

Table 2. Test for normality (Shapiro-Wilk test)

Site	Narela		Okhla		Pusa	
Series	$PM_{2.5}$	PM_{10}	$PM_{2.5}$	PM_{10}	$PM_{2.5}$	PM_{10}
Test statistic	0.857	0.950	0.813	0.937	0.832	0.955
p-value	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001

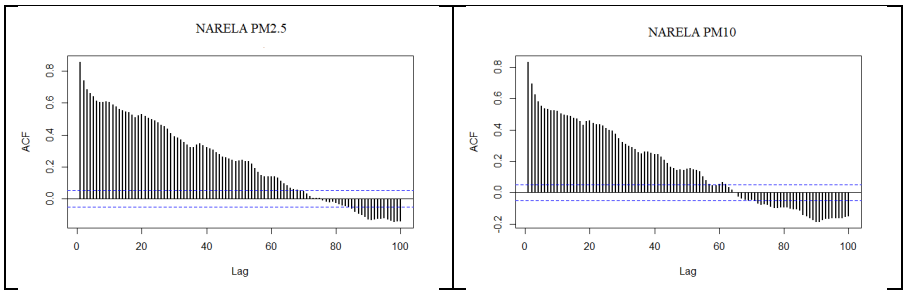
The stationarity of the price return series is tested using Augmented Dickey-Fuller (ADF) test [16] and Phillips-Perron (PP) test [17]. For ADF and PP tests, the null hypothesis is that series is not stationary. Both the tests confirm (Table 3) that all the selected series are stationary.

Table 3. Test for stationarity of the selected series

Site	Narela		Okhla		Pusa	
Test	$PM_{2.5}$	PM_{10}	$PM_{2.5}$	PM_{10}	$PM_{2.5}$	PM_{10}
ADF	-3.922 (0.01)	-4.429 (0.01)	-3.654 (0.03)	-3.944 (0.01)	-3.587 (0.03)	-3.829 (0.02)
PP	-9.686 (0.01)	-10.890 (0.01)	-9.468 (0.01)	-10.307 (0.01)	-8.811 (0.01)	-9.449 (0.01)

p-value is in parenthesis

The statistical dependencies among the realizations of a time series can be inspected by using the ACF and PACF plots. The ACF plots (up to 100 lags) of the selected series are given in Figure 2. From this figure, it can be seen that the ACFs are significant for a large lags. This is known as the hyperbolic decay of ACF. This clearly indicates the presence of long memory in the mean structure. Hence, the ARFIMA model is applied to the data set, instead of the ARIMA model.



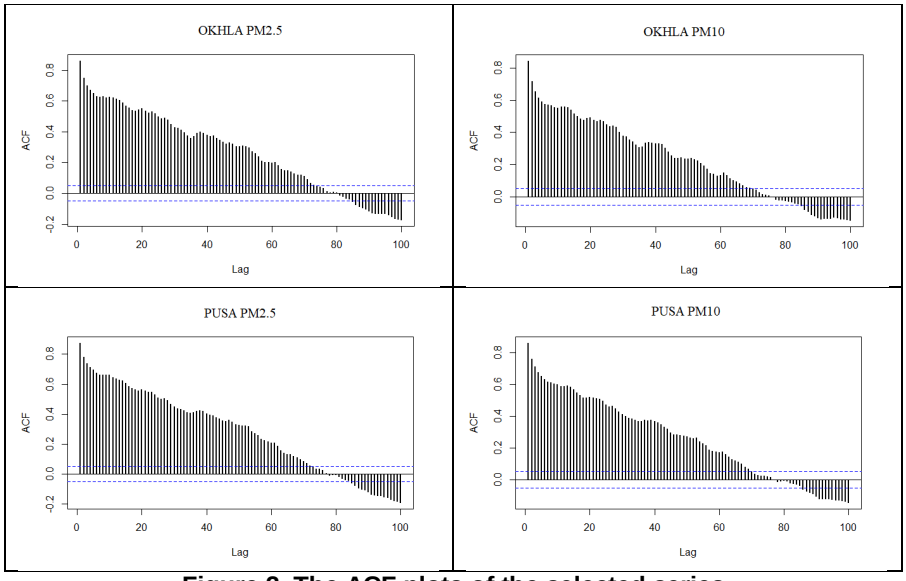


Figure 2. The ACF plots of the selected series

Various orders of autoregression and moving average of the ARFIMA (p, d, q) model are applied to each series. The residuals are obtained and are tested using the ARCH-LM test for the potential presence of conditional heteroscedasticity. For each instances, the ARCH-LM test results affirmative. After that the GARCH(1,1) model is fitted to the residual series and the best performed order of autoregression and moving average is chosen based on minimum values of Akaike Information Criterion (AIC) and Bayesian information Criterion (BIC). The estimated parameters of the selected ARFIMA (p, d, q)-GARCH(1,1) models are given in Table 4. It can be seen that almost all the estimated parameters are significant at 1% level of significance. The visualization of observed vs. fitted values is given in Figure 3. The residual series are tested for the presence variability that can be explained further. It is seen that all the residual series are white noise (WN). This proved that the selected ARFIMA (p, d, q)- GARCH(1,1) model is an appropriate model for forecasting the data under study. The forecast efficiency of the selected models is validated (Table 5) in the model validation set using three error functions namely RMSE, MAE and MAPE. It is seen that the values of these error functions are within permissible limit. Hence, the performance of the selected models is satisfactory.

Table 4. Estimate of parameters of the selected models

	Narela		Okhla		Pusa	
	$PM_{2.5}$	PM_{10}	$PM_{2.5}$	PM_{10}	$PM_{2.5}$	PM_{10}
Model / Variable	ARFIMA (1,d,2) - GARCH (1,1)	ARFIMA (2,d,2) - GARCH (1,1)	ARFIMA (1,d,2) - GARCH (1,1)	ARFIMA (1,d,2) - GARCH (1,1)	ARFIMA (1,d,2) - GARCH (1,1)	ARFIMA (2,d,1) - GARCH (1,1)
Mean Model						
Constant	34.770 (3.622)***	103.945 (12.221)***	26.059 (2.579)***	100.803 (7.831)***	34.451 (2.702)***	126.451 (4.495)***
AR(1)	0.987 (0.015)***	1.411 (0.009)***	0.990 (0.005)***	0.983 (0.011)***	0.988 (0.005)***	1.346 (0.017)***
AR(2)		-0.419 (0.008)***				-0.356 (0.005)***
MA(1)	-0.703	-0.762	-0.598	-0.660	-0.659	-0.930

	(0.057)***	(0.045)***	(0.013)***	(0.032)***	(0.018)***	(0.052)***
MA(2)	-0.236 (0.024)***	-0.125 (0.022)***	-0.323 (0.014)***	-0.276 (0.016)***	-0.266 (0.018)***	
d	0.459 (0.131)***	0.147 (0.042)***	0.386 (0.029)***	0.485 (0.055)***	0.415 (0.032)***	0.391 (0.039)***
Variance Model						
Constant	12.430 (4.407)***	46.669 (21.206)**	17.645 (4.937)***	73.193 (27.636)***	14.947 (4.476)***	62.191 (21.912)***
α_1	0.147 (0.021)***	0.108 (0.018)***	0.205 (0.029)***	0.161 (0.027)***	0.211 (0.033)***	0.145 (0.028)***
β_1	0.852 (0.019)***	0.891 (0.017)***	0.794 (0.025)***	0.838 (0.023)***	0.788 (0.028)***	0.853 (0.023)***

***p<0.01, **p<0.05, *p<0.10; S.E. is in parenthesis

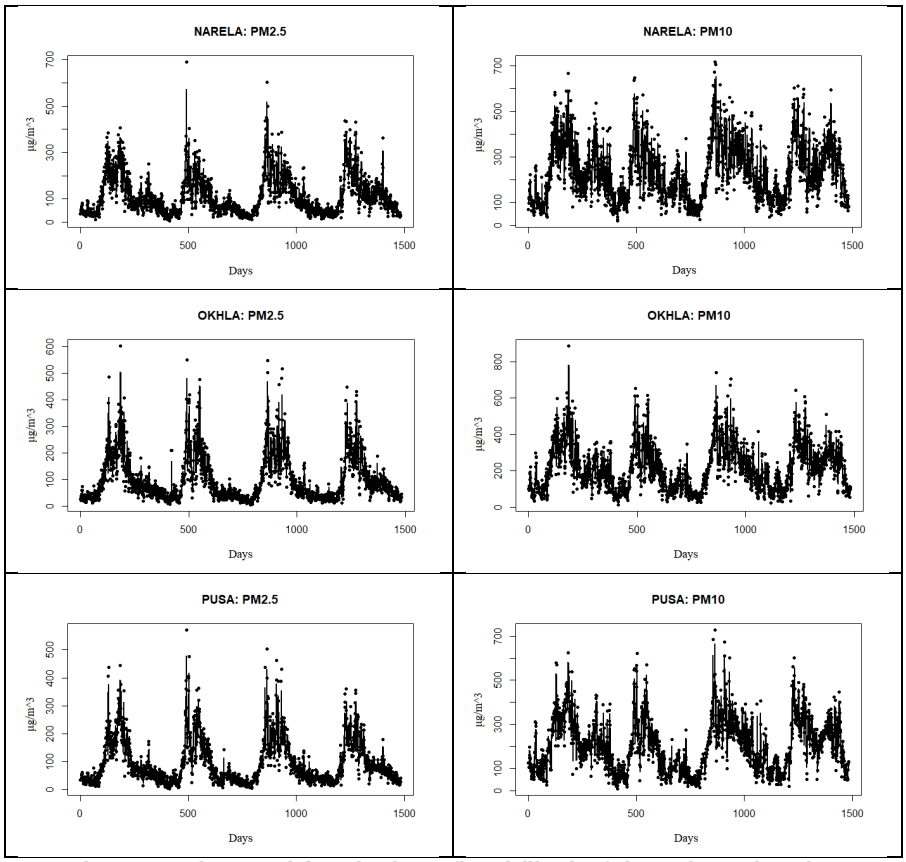


Figure 3. Observed (marker) vs. fitted (line) of the selected series

Table 5. Forecasting performance of fitted models at model validation set

Site		Model	RMSE	MAE	MAPE (%)
Narela	$PM_{2.5}$	ARFIMA (1,d,2) -GARCH (1,1)	15.411	14.045	8.268
	PM_{10}	ARFIMA (2,d,2) -GARCH(1,1)	40.290	35.496	6.779
Okhla	$PM_{2.5}$	ARFIMA (1,d,2) -GARCH (1,1)	16.834	16.045	8.639
	PM_{10}	ARFIMA (1,d,2) -GARCH (1,1)	41.775	39.029	6.428

Pusa	$PM_{2.5}$	ARFIMA (1,d,2) -GARCH (1,1)	14.677	14.128	7.928
	PM_{10}	ARFIMA (2,d,1) -GARCH (1,1)	50.594	44.329	8.773

4. CONCLUSION

In this paper, an attempt has been made to model the variation of $PM_{2.5}$ and PM_{10} concentration for three selected location in Delhi-NCR. Due to the presence of long memory in mean structure, the ARFIMA model is applied as the mean model and the GARCH model as the variance model. Modeling can be helpful for understanding the fluctuation of concentrate of these two pollutants. Apart from the year-round measures already in place, the peak period of air pollution in Delhi, from late-October to early-January, demands special attention of policymakers. Few of the proactive measures that can substantially alleviate the PM-pollution are banning the burning of stubbles and firecrackers, installation of smog towers, and being more stringent about vehicular emissions.

REFERENCES

[1]. Das M, Das A, Sarkar R, Mandal P, Saha S, Ghosh S. Exploring short term spatio-temporal pattern of $PM_{2.5}$ and PM_{10} and their relationship with meteorological parameters during COVID-19 in Delhi. *Urban Climate* 2021;39:100944.

[2]. Vohra K, Vodonos A, Schwartz J, Marais EA, Sulprizio MP, Mickley LJ. Global mortality from outdoor fine particle pollution generated by fossil fuel combustion: Results from GEOS-Chem. *Environmental Research* 2021;195:110754.

[3]. Dimitrova A, Marois G, Kiesewetter G, Samir KC, Rafaj P, Tonne C. Health impacts of fine particles under climate change mitigation, air quality control, and demographic change in India. *Environmental Research Letters* 2021;16(5):054025.

[4]. IQAir. World's most polluted countries and regions (historical data 2018–2021). 2021a. Available in https://www.iqair.com/world-most-polluted-countries. (Last accessed on 14th July, 2022)

[5]. WHO. Ambient (outdoor) air pollution. 2021. Available in https://www.who.int/news-room/fact-sheets/detail/ambient-(outdoor)-air-quality-and-health. (Last accessed on 14th July, 2022)

[6]. Institute for Health Metrics and Evaluation (IHME). What risk factors drive the most death and disability combined? 2019. Available in https://www.healthdata.org/india. (Last accessed on 14th July, 2022)

[7]. Pandey A, Brauer M, Cropper ML, Balakrishnan K, Mathur P, Dey S, Turkgulu B, Kumar GA, Khare M, Beig G, Gupta T. Health and economic impact of air pollution in the states of India: the Global Burden of Disease Study 2019. *The Lancet Planetary Health* 2021;5(1):25–38.

[8]. IQAir. World's most polluted cities (historical data 2017–2021). 2021b. Available in https://www.iqair.com/in-en/world-most-polluted-cities. (Last accessed on 14th July, 2022)

[9]. Box GEP, Jenkins G. Time Series Analysis, Forecasting and Control; Holden-Day: San Francisco, CA, USA, 1970.

[10]. Granger CWJ, Joyeux R. An introduction to long-memory time series models and fractional differencing. *Journal of Time Series Analysis* 1980;4:221–238.

[11]. Geweke J, Porter-Hudak S. The estimation and application of long-memory time series models. *Journal of Time Series Analysis* 1983;4:221–238.

[12]. Engle RF. Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica* 1982; 50(4):987–1007.

[13]. Bollerslev T. Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics* 1986;31(3):307–327.

[14]. Hosking JRM. Fractional differencing. *Biometrika* 1981;68:165–176.

[15]. Shapiro SS, Wilk MB. An analysis of variance test for normality (complete samples). *Biometrika* 1965;52(3/4):591–611.

[16]. Dickey D, Fuller W. Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of American Statistical Association* 1979;74:427–431.
[17]. Phillips PCB, Perron P. Testing for a unit root in time series regression. *Biometrika* 1988;75(2):335–346.