Original Research Article

On Neutrosophic Z-algebras

Abstract

This study presents the notion of neutrosophicZ-algebra and neutrosophic pseudo Z-algebra explores some of its properties. Also studied are the neutrosophic Z-ideal, neutrosophic Z-sub algebra, and neutrosophic Z-filter. Several properties are discovered, and some findings from the study of homomorphism are discussed.

Keywords:neutrosophicZ-algebra,neutrosophic pseudo Z-algebra ,neutrosophicZ-sub algebra,neutrosophicZ-ideal,neutrosophicZ-filter

1.Introduction:

Smarandache established the area of philosophy known as neutrosophy, which has a manyimplementations in the real world and in mathematics, particularly in algebra[1].also gave more informationabout neutrosophysee[2,3].making use of neutrosophic theory

Kandasamy and Smarandache[4] in 2004suggested a set-based algebraic structureof neutrosophic numbers of the type $\mathcal{N} = \mathcal{Z} + \dagger\mathcal{I}$ that they dubbed $\mathcal{I}$-Neutrosophic Algebraic Structure.,where $\mathcal{Z}, \dagger \in \mathbb{R}\ or\ \mathbb{C}$, and $\mathcal{I}$ which means indetermined or uncertain thus that$\mathcal{I}^2 = \mathcal{I}$,is referred to as literal indeterminacy, here$\mathcal{Z}$ is referred to as the $\mathcal{N}'$s determinate portion, and $\dagger\mathcal{I}$ is referred to as its indeterminate portionon $\mathcal{N}$,with $g\mathcal{I} + h\mathcal{I} = (g + h)\mathcal{I}$, $0.\mathcal{I} = 0$. Where$\mathcal{I}$ is different from the imaginary$i^2 = -1$,in general, $\mathcal{I}^{j} = \mathcal{I}\ if\ \ j > 0$,and is unknown for$j \leq 0$. In 2006, the idea of neutrosophic algebraic structures was also proposed [5].

In [6,7,8,9], the idea of neutrosophic BCI/BCK –algebras,neutrosophic KU-algebras and neutrosophic B-algebras was presented.

Z-algebra is an unique algebraic structure basedon logic that was first proposed in 2017 by Chandramouleeswaran et al. [10].

[11] and [12]They provided characteristics and further explanation ofZ-algebra.

In this article, we explain the idea of neutrosophic Z-algebra, look at various relevant characteristics, examine a neutrosophic Z-homomorphism, and present some findings.

2. preliminaries:

Definition2.1: [1] A neutronsophicset $\mathcal{X}(\mathcal{I}) = \langle \mathcal{X}, \mathcal{I} \rangle = \{ϩ + ϯ\mathcal{I} : ϩ, ϯ \in \mathcal{X}\}$, where $\mathcal{X} \neq \phi$ and $\mathcal{I}$ an indeterminate.

Definition2.2: [10] let $\mathcal{Z} \neq \phi$ and $*$ is a binary operation with constant 0 then the algebra $(\mathcal{Z}, *, 0)$ named Z-Algebra if satisfying the following axiom:

$\mathcal{Z}_1$: $ϩ * 0 = 0$

$\mathcal{Z}_2$: $0 * ϩ = ϩ$

$\mathcal{Z}_3$: $ϩ * ϩ = ϩ$

$\mathcal{Z}_4$: $ϩ * ϯ = ϯ * ϩ$ When $ϩ \neq 0$ and $ϯ \neq 0, \forall ϩ, ϯ \in \mathcal{Z}$.

Definition2.3:[10] Let $\delta \neq \phi$ and $\delta \subseteq \mathcal{Z}$ where $(\mathcal{Z}, *, 0)$ is aZ-Algebra, δ is named Z-subalgebraif $ϩ * ϯ \in \delta$, $\forall ϩ, ϯ \in \delta$.

Definition2.4:[10] Let $\mathcal{I} \neq \phi$ and $\mathcal{I} \subseteq \mathcal{Z}$,where $(\mathcal{Z}, *, 0)$is a Z-Algebra,$\mathcal{I}$ is named Z-ideal of $\mathcal{Z}$ if satisfy(1) $0 \in \mathcal{I}$(2) $ϩ * ϯ \in \mathcal{I}$, and $ϯ \in \mathcal{I} \Rightarrow ϩ \in \mathcal{I}$.

Definition2.5:[11] Let $\mathcal{I} \neq \phi$ and $\mathcal{I} \subseteq \mathcal{Z}$,where $(\mathcal{Z}, *, 0)$is a Z-Algebra,$\mathcal{I}$ is named Z_1-ideal of $\mathcal{Z}$ if satisfy

(1) $0 \in \mathcal{I}$ (2) $((ϩ * ϡ) * ϩ) * ϯ \in \mathcal{I}$, and $ϯ \in \mathcal{I} \Rightarrow ϩ \in \mathcal{I}, \forall ϩ, ϯ, ϡ \in \mathcal{Z}$.

Definition2.6: [11] Let $\mathcal{I} \neq \phi$ and $\mathcal{I} \subseteq \mathcal{Z}$,where $(\mathcal{Z}, *, 0)$is a Z-Algebra,$\mathcal{I}$ is named Z_2-ideal of $\mathcal{Z}$ if satisfy

(1) $0 \in \mathcal{I}$ (2) $(ϩ * ϡ) * (ϩ * ϯ) \in \mathcal{I}$, and $ϯ \in \mathcal{I} \Rightarrow ϩ \in \mathcal{I}, \forall ϩ, ϯ, ϡ \in \mathcal{Z}$

Definition2.7:[12] Let $\mathcal{I} \neq \phi$ and $\mathcal{I} \subseteq \mathcal{Z}$,where $(\mathcal{Z}, *, 0)$is a Z-Algebra,$\mathcal{I}$ is named Z_P-ideal of $\mathcal{Z}$ if satisfy

(1) $0 \in \mathcal{I}$ (2) $(ϩ * ϡ) * (ϯ * ϡ) \in \mathcal{I}$, and $ϯ \in \mathcal{I} \Rightarrow ϩ \in \mathcal{I}, \forall ϩ, ϯ, ϡ \in \mathcal{Z}$.

Definition2.8:[10] let $\mathcal{F} \neq \phi$ and $\mathcal{F} \subseteq \mathcal{Z}$,where $(\mathcal{Z}, *, 0)$is a Z-Algebra,$\mathcal{F}$is named $\mathcal{Z}$-filter of $\mathcal{Z}$ if $ϩ ⅄ ϯ = ϩ * (ϩ * ϯ) \in \mathcal{F}, \forall ϩ, ϯ \in \mathcal{F}, (ϩ \neq ϯ)$.

Example2.9: let $\mathcal{Z} = \{0, ϩ, ϯ, ϡ\}$ be set and $*$ is a binary operation de ined on $\mathcal{Z}$ by the table:

$*$	**0**	ϩ	ϯ	ϡ
0	0	ϩ	ϯ	ϡ
ϩ	0	ϩ	ϯ	ϯ
ϯ	0	ϯ	ϯ	ϯ
ϡ	0	ϯ	ϯ	ϡ

Then $(\mathcal{Z},*,0)$ is Z-Algebra $.\delta = \{ϩ, ϯ, ঌ\}$ is Z-subalgebra and $\mathcal{I} = \{0, ϩ, ϯ\}$ is a Z_1 –ideal, $\mathcal{I}^{\$} = \{0, ϩ, ঌ\}$ is a (Z_2 -ideal ,Z_p –ideal) Z-ideal .and $\mathcal{F} = \{ϩ, ϯ\}$ is $\mathcal{Z}$-filter .

Note: every (Z_1– ideal , Z_2 –ideal) is an ideal of $\mathcal{Z}$.

Definition2.10: [11] Let $\mathcal{Z} \neq \phi$ with two binary operations $*, \circledast$ and constant 0 then the algebra $(\mathcal{Z},*,\circledast,0)$ named pseudo Z-Algebra(briefly,$P\mathcal{Z}$) if satisfying the following axiom:

$P\mathcal{Z}_1$: $ϩ * 0 = ϩ \circledast 0 = 0$

$P\mathcal{Z}_2$: $0 * ϩ = 0 \circledast ϩ = ϩ$

$P\mathcal{Z}_3$: $ϩ * ϩ = ϩ \circledast ϩ = ϩ$

$P\mathcal{Z}_4$: $ϩ * ϯ = ϯ \circledast ϩ$ When $ϩ \neq 0$ and $ϯ \neq 0, \forall\ ϩ, ϯ \in \mathcal{Z}$.

Definition2.11: [11] Let $\delta \neq \phi$ and $\delta \subseteq \mathcal{Z}$,where $(\mathcal{Z},*,\circledast,0)$ is $P\mathcal{Z}$ then δ is named a pseudo Z-subalgebraif $ϩ * ϯ,\ ϩ \circledast ϯ \in \delta, \forall\, ϩ, ϯ \in \delta$.

Example2.12:Let $\mathcal{Z} = \{0, ϩ, ϯ, ঌ\}$ be setand $*, \circledast$ are a binary operations de ined on $\mathcal{Z}$ bythe table as follows:

$*$	0	ϩ	ϯ	ঌ	$\circledast$	0	ϩ	ϯ	ঌ
0	0	ϩ	ϯ	ঌ	0	0	ϩ	ϯ	ঌ
ϩ	0	ϩ	ϩ	ϯ	ϩ	0	ϩ	ϯ	ϩ
ϯ	0	ϯ	ϯ	ϩ	ϯ	0	ϩ	ϯ	ϩ
ঌ	0	ϩ	ϩ	ঌ	ঌ	0	ϯ	ϩ	ঌ

Then $(\mathcal{Z},*,\circledast,0)$ is pseudo Z-algebra ,$\delta = \{ϩ, ϯ, ঌ\}$ is apseudo Z-sub algebra.

3.Neutrosophic Z-algebra:

Definition 3.1: A neutrosophic Z-algebra is the triple$\big(\mathcal{Z}(\mathcal{I}),*,(0,0\mathcal{I})\big)$(briefly,$\mathcal{N}\mathcal{Z}$)(where$(\mathcal{Z},*,0)$be a Z-algebra,$\mathcal{Z}(\mathcal{I}) = \langle \mathcal{Z}, \mathcal{I} \rangle$ a neutrosophic set)

if$(ϩ, ƅ\mathcal{I}), (ϯ, Ч\mathcal{I})$ are any two elements of $\mathcal{Z}(\mathcal{I})$ with $ϩ, ƅ, ϯ, Ч \in \mathcal{Z}$ satisfies

$$(ϩ, ƅ\mathcal{I}) * (ϯ, Ч\mathcal{I}) = (ϩ * ϯ, (ϩ * Ч \wedge ƅ * ϯ \wedge ƅ * Ч)\mathcal{I})$$

An element $ϩ \in \mathcal{Z}$ is represented by $(ϩ, 0\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$,

$(ϩ, 0\mathcal{I}) * (ƅ, 0\mathcal{I}) = (ϩ * ƅ, 0\mathcal{I}) = (ϩ \wedge \backsim ƅ, 0)$. where $\backsim ƅ$ is the negation of $ƅ$ in$\mathcal{Z}$

And $(ϩ, ƅ\mathcal{I}) = (ϯ, Ч\mathcal{I}) \Leftrightarrow (ϩ = ϯ$ and $ƅ = Ч)$

Definition 3.2: A neutrosophicpseudoZ-algebra is $(\mathcal{Z}(\mathcal{I}),*,\circledast,(0,0\mathcal{I}))$(briefly,$\mathcal{N}P\mathcal{Z}$) (where$(\mathcal{Z},*,\circledast,0)$be a pseudo Z-algebra

If $(\mathfrak{Z}, \mathfrak{b}\mathcal{I}), (\dagger, \mathfrak{q}\mathcal{I})$ are any two elements of $\mathcal{Z}(\mathcal{I})$ with $\mathfrak{x}, \mathfrak{b}, \dagger, \mathfrak{q} \in \mathcal{Z}$ satisfies

$(\mathfrak{Z}, \mathfrak{b}\mathcal{I}) * (\dagger, \mathfrak{q}\mathcal{I}) = (\mathfrak{Z} * \dagger, (\mathfrak{Z} * \mathfrak{q} \wedge \mathfrak{b} * \dagger \wedge \mathfrak{b} * \mathfrak{q})\mathcal{I})$

$(\dagger, \mathfrak{q}\mathcal{I}) \circledast (\mathfrak{Z}, \mathfrak{b}\mathcal{I}) = (\dagger \circledast \mathfrak{Z}, (\mathfrak{Z} \circledast \mathfrak{q} \wedge \mathfrak{b} \circledast \dagger \wedge \mathfrak{b} \circledast \mathfrak{q})\mathcal{I})$

Where$(\mathfrak{Z}, \mathfrak{b}\mathcal{I}) * (\dagger, \mathfrak{q}\mathcal{I}) = (\mathfrak{Z}, \mathfrak{b}\mathcal{I}) \circledast (\dagger, \mathfrak{q}\mathcal{I})$When$(\mathfrak{Z}, \mathfrak{b}\mathcal{I}) \neq (0,0\mathcal{I})$ and $(\dagger, \mathfrak{q}\mathcal{I}) \neq (0,0\mathcal{I}), \forall (\mathfrak{Z}, \mathfrak{b}\mathcal{I}), (\dagger, \mathfrak{q}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$

Theorem 3.3: Every $\mathcal{NZ}\big(\mathcal{Z}(\mathcal{I}),*, (0,0\mathcal{I})\big)$ with condition$(0,0\mathcal{I}) * (\mathfrak{Z}, \mathfrak{b}\mathcal{I}) = (\mathfrak{Z}, \mathfrak{b}\mathcal{I})$ is a$\mathcal{Z}$–algebra and conversely, not.

Proof:let$\big(\mathcal{X}(\mathcal{I}),*, (0,0\mathcal{I})\big)$ is $\mathcal{NZ}$

Let $r = (\mathfrak{Z}, \mathfrak{b}\mathcal{I})\ and\ 0 = (0,0\mathcal{I})$

$\mathcal{Z}_1$: $r * 0 = (\mathfrak{Z}, \mathfrak{b}\mathcal{I}) * (0,0\mathcal{I}) = (\mathfrak{Z} * 0, (\mathfrak{Z} * 0 \wedge \mathfrak{b} * 0)\mathcal{I}) = (0, (0 \wedge 0)\mathcal{I}) = (0,0\mathcal{I})$

$\mathcal{Z}_2$: $0 * r = (0,0\mathcal{I}) * (\mathfrak{Z}, \mathfrak{b}\mathcal{I}) = (0 * \mathfrak{Z}, (0 * \mathfrak{b} \wedge 0 * \mathfrak{Z})\mathcal{I}) = (\mathfrak{Z}, (\mathfrak{b} \wedge \mathfrak{Z})\mathcal{I}) = (\mathfrak{Z}, \mathfrak{b}\mathcal{I})$

$\mathcal{Z}_3$: $r * r = (\mathfrak{Z}, \mathfrak{b}\mathcal{I}) * (\mathfrak{Z}, \mathfrak{b}\mathcal{I}) = (\mathfrak{Z} * \mathfrak{Z}, (\mathfrak{Z} * \mathfrak{b} \wedge \mathfrak{b} * \mathfrak{Z} \wedge \mathfrak{b} * \mathfrak{b})\mathcal{I})$

$=(\mathfrak{Z}, (\mathfrak{Z} \wedge \backsim \mathfrak{b} \wedge \mathfrak{b} \wedge \backsim \mathfrak{Z} \wedge \mathfrak{b})\mathcal{I})$

$=(\mathfrak{Z}, \mathfrak{b}\mathcal{I})$

$\mathcal{Z}_4$: $if\ r * s = s * r$, when $r \neq 0 \& s \neq 0, \forall\ r, s \in \mathcal{Z}(\mathcal{I})$

$let\ r = (\mathfrak{Z}, \mathfrak{b}\mathcal{I}), s = (\dagger, \mathfrak{q}\mathcal{I})$,

$$(\mathfrak{Z}, \mathfrak{b}\mathcal{I}) * (\dagger, \mathfrak{q}\mathcal{I}) = (\dagger, \mathfrak{q}\mathcal{I}) * (\mathfrak{Z}, \mathfrak{b}\mathcal{I})$$

$(\mathfrak{Z} * \dagger, (\mathfrak{Z} * \mathfrak{q} \wedge \mathfrak{b} * \dagger \wedge \mathfrak{b} * \mathfrak{q})\mathcal{I}) = (\dagger * \mathfrak{Z}, (\dagger * \mathfrak{b} \wedge \mathfrak{q} * \mathfrak{Z} \wedge \mathfrak{q} * \mathfrak{b})\mathcal{I})$

Suppose $(\mathfrak{Z}, \mathfrak{b}\mathcal{I}) \neq (0,0\mathcal{I}) \& (\dagger, \mathfrak{q}\mathcal{I}) \neq (0,0\mathcal{I})$we get

$0 * \dagger = \dagger * 0 \quad \Rightarrow \dagger = 0$

and $0 * \mathfrak{q} \wedge 0 * 0 = 0 * 0 \wedge \mathfrak{q} * 0 \ \Rightarrow\ \mathfrak{q} = 0$

We get a contradiction.

Then $\big(\mathcal{Z}(\mathcal{I}),*, (0,0\mathcal{I})\big)$is a Z-algebra.

Theorem 3.4: Every $\mathcal{N}\mathrm{P}\mathcal{Z}$, $(\mathcal{Z}(\mathcal{I}),*,\circledast, (0,0\mathcal{I}))$ with condition$(0,0\mathcal{I}) * (\mathfrak{Z}, \mathfrak{b}\mathcal{I}) = (\mathfrak{Z}, \mathfrak{b}\mathcal{I}), (0,0\mathcal{I}) \circledast (\mathfrak{Z}, \mathfrak{b}\mathcal{I}) = (\mathfrak{Z}, \mathfrak{b}\mathcal{I})$ is a pseudo$\mathcal{Z}$ -algebra and conversely, not.

Proof: it is easy as above.

Definition 3.5: Let$\mathfrak{S}(\mathcal{I}) \neq \phi$ and $\mathfrak{S}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$,$\big(\mathcal{Z}(\mathcal{I}),*, (0,0\mathcal{I})\big)$is $\mathcal{NZ}$,$\mathfrak{S}(\mathcal{I})$ is named a neutrosophicZ-subalgebra(briefly, $\mathcal{NZ}^{s}$) of $\mathcal{Z}(\mathcal{I})$ if

1) $(0,0\mathcal{I}) \in \mathfrak{S}(\mathcal{I})$

2) $(Ƨ , ƀ\mathcal{J}) * († , Ч\mathcal{J}) \in \mathfrak{S}(\mathcal{J}), \forall (Ƨ , ƀ\mathcal{J}), († , Ч\mathcal{J}) \in \mathfrak{S}(\mathcal{J})$
3) $\mathfrak{S}(\mathcal{J})$Contains a proper sub set which a Z-algebra.

Definition 3.6: Let$\mathfrak{S}(\mathcal{J}) \neq \phi$ and $\mathfrak{S}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$,$(\mathcal{Z}(\mathcal{J}),*,\circledast, (0,0\mathcal{J})is\mathcal{N}\mathrm{P}\mathcal{Z}$, $\mathfrak{S}(\mathcal{J})$ is called a neutrosophicpseudoZ-subalgebra(briefly, $\mathcal{N}\mathrm{P}\mathcal{Z}^{\mathcal{s}}$) of $\mathcal{Z}(\mathcal{J})$ if

1) $(0,0\mathcal{J}) \in \mathfrak{S}(\mathcal{J})$
2) $(Ƨ , ƀ\mathcal{J}) * († , Ч\mathcal{J}) \in \mathfrak{S}(\mathcal{J})\&(Ƨ , ƀ\mathcal{J}) \circledast († , Ч\mathcal{J}) \in \mathfrak{S}(\mathcal{J}), \forall (Ƨ , ƀ\mathcal{J}), († , Ч\mathcal{J}) \in \mathfrak{S}(\mathcal{J})$
3) $\mathfrak{S}(\mathcal{J})$Contains a proper sub set which a pseudo Z-algebra.

Theorem 3.7: If$\mathcal{A}_{(ω,ω\mathcal{J})}(\mathcal{J}) \neq \phi$ and $\mathcal{A}_{(ω,ω\mathcal{J})}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$ for $ω \neq 0$,$\big(\mathcal{Z}(\mathcal{J}),*, (0,0\mathcal{J})\big)$ is $\mathcal{N}\mathcal{Z}$**,**where$\mathcal{A}_{(ω,ω\mathcal{J})}(\mathcal{J}) = \{(Ƨ , ƀ\mathcal{J}) \in \mathcal{Z}(\mathcal{J}) : (Ƨ , ƀ\mathcal{J}) * (ω, ω\mathcal{J}) = (ω, ω\mathcal{J})\}$

Then 1) $\mathcal{A}_{(ω,ω\mathcal{J})}(\mathcal{J})$ is $\mathcal{N}\mathcal{Z}^{\mathcal{s}}$.

2) $\mathcal{A}_{(ω,ω\mathcal{J})}(\mathcal{J}) \subseteq \mathcal{A}_{(0,0I)}(\mathcal{J})$.

Proof:1) clearly$(0,0\mathcal{J}) \in \mathcal{A}_{(ω,ω\mathcal{J})}(\mathcal{J})$

$\mathcal{A}_{(ω,ω\mathcal{J})}(\mathcal{J})$contain a proper sub setwhich a Z-algebra.

Let $(Ƨ , ƀ\mathcal{J}), († , Ч\mathcal{J}) \in \mathcal{A}_{(ω,ω\mathcal{J})}(\mathcal{J}) \Rightarrow$

$(Ƨ , ƀ\mathcal{J}) * (ω, ω\mathcal{J}) = (ω, ω\mathcal{J})$, $(† , Ч\mathcal{J}) * (ω, ω\mathcal{J}) = (ω, ω\mathcal{J}) \Rightarrow$

$Ƨ * ω = ω$, $Ƨ * ω \wedge ƀ * ω = ω$ &$† * ω = ω$, $† * ω \wedge Ч * ω = ω$since$ω \neq 0 \Rightarrow$

$$Ƨ = ƀ = † = Ч = ω$$

$$[(Ƨ , ƀ\mathcal{J}) * († , Ч\mathcal{J})] * (ω, ω\mathcal{J}) = [Ƨ * †, (Ƨ * Ч \wedge ƀ * †)\mathcal{J}] * (ω, ω\mathcal{J})$$

$$=\big[(Ƨ * †) * ω , \big((Ƨ * †) * ω \wedge (Ƨ * Ч \wedge ƀ * †) * ω\big)\mathcal{J}\big]$$

$$=[ω * ω, (ω * ω \wedge ω * ω)\mathcal{J}]$$

$$= (ω, ω\mathcal{J})$$

This shows that $(Ƨ , ƀ\mathcal{J}) * († , Ч\mathcal{J}) \in \mathcal{A}_{(ω,ω\mathcal{J})}(\mathcal{J})$

Then$\mathcal{A}_{(ω,ω\mathcal{J})}(\mathcal{J})$is $\mathcal{N}\mathcal{Z}^{\mathcal{s}}$.

(2) it's easy.

Theorem 3.8: If $\mathcal{A}_{(ω,ω\mathcal{J})}(\mathcal{J}) \neq \phi$ and $\mathcal{A}_{(ω,ω\mathcal{J})}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, for $ω \neq 0$,

$(\mathcal{Z}(\mathcal{J}),*,\circledast, (0,0\mathcal{J})$ is$\mathcal{N}\mathrm{P}\mathcal{Z}$**,**where$\mathcal{A}_{(ω,ω\mathcal{J})}(\mathcal{J}) = \{(Ƨ , ƀ\mathcal{J}) \in \mathcal{Z}(\mathcal{J}) : (Ƨ , ƀ\mathcal{J}) * (ω, ω\mathcal{J}) = (ω, ω\mathcal{J})\&(Ƨ , ƀ\mathcal{J}) \circledast (ω, ω\mathcal{J}) = (ω, ω\mathcal{J})\}$

Then 1) $\mathcal{A}_{(ω,ω\mathcal{J})}(\mathcal{J})$ is $\mathcal{N}\mathrm{P}\mathcal{Z}^{\mathcal{s}}$.

2) $\mathcal{A}_{(ω,ω\mathcal{J})}(\mathcal{J}) \subseteq \mathcal{A}_{(0,0\mathcal{J})}(\mathcal{J})$.

Proof: it is easy as above.

Theorem 3.9: If$\mathcal{Z}_\xi(\mathcal{J}) \neq \phi$ and $\mathcal{Z}_\xi(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$,$(\mathcal{Z}(\mathcal{J}),*,(0,0\mathcal{J}))$ is $\mathcal{NZ}$**,**where

$\mathcal{Z}_\xi(\mathcal{J}) = \{(Ƨ,Ƨ\mathcal{J}) : Ƨ \in \mathcal{Z}\}$Then$\mathcal{Z}_\xi(\mathcal{J})isa\mathcal{NZ}^sof\mathcal{Z}(\mathcal{J})$.

Proof:clearly$(0,0\mathcal{J}) \in \mathcal{Z}_\xi(\mathcal{J})$and the third condition is satisfied for $\mathcal{Z}_\xi(\mathcal{J})$

Let $(†,†\mathcal{J}),(ƀ,ƀ\mathcal{J}) \in \mathcal{Z}_\xi(\mathcal{J}),\ †,ƀ \in \mathcal{Z} \Rightarrow$

$(†,†\mathcal{J}) * (ƀ,ƀ\mathcal{J}) = († * ƀ,(† * ƀ)\mathcal{J})$

This shows that $(†,†\mathcal{J}) * (ƀ,ƀ\mathcal{J}) \in \mathcal{Z}_\xi(\mathcal{J})$

Then $\mathcal{Z}_\xi(\mathcal{J})$is a $\mathcal{NZ}^s$of$\mathcal{Z}(\mathcal{J})$.

Theorem 3.10: If$\mathcal{Z}_\xi(\mathcal{J}) \neq \phi$ and $\mathcal{Z}_\xi(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$,$(\mathcal{Z}(\mathcal{J}),*,\circledast,(0,0\mathcal{J})$ is$\mathcal{N}\mathrm{P}\mathcal{Z}$ **,**where

$\mathcal{Z}_\xi(\mathcal{J}) = \{(Ƨ,Ƨ\mathcal{J}) : Ƨ \in \mathcal{Z}\}$ Then $\mathcal{Z}_\xi(\mathcal{J})$is a$\mathcal{N}\mathrm{P}\mathcal{Z}^sof\mathcal{Z}(\mathcal{J})$.

Proof: it is easy as above.

Example3.11: Let$*$is a binary operation defined on

$\mathcal{Z}_\xi(\mathcal{J}) = \{(0,0\mathcal{J}),(Ƨ,Ƨ\mathcal{J}),(†,†\mathcal{J}),(ঽ,ঽ\mathcal{J})\}$as follows:

$*$	$(0,0\mathcal{J})$	$(Ƨ,Ƨ\mathcal{J})$	$(†,†\mathcal{J})$	$(ঽ,ঽ\mathcal{J})$
$(0,0\mathcal{J})$	$(0,0\mathcal{J})$	$(Ƨ,Ƨ\mathcal{J})$	$(†,†\mathcal{J})$	$(ঽ,ঽ\mathcal{J})$
$(Ƨ,Ƨ\mathcal{J})$	$(0,0\mathcal{J})$	$(Ƨ,Ƨ\mathcal{J})$	$(0,0\mathcal{J})$	$(Ƨ,Ƨ\mathcal{J})$
$(†,†\mathcal{J})$	$(0,0\mathcal{J})$	$(0,0\mathcal{J})$	$(†,†\mathcal{J})$	$(†,†\mathcal{J})$
$(ঽ,ঽ\mathcal{J})$	$(0,0\mathcal{J})$	$(Ƨ,Ƨ\mathcal{J})$	$(†,†\mathcal{J})$	$(ঽ,ঽ\mathcal{J})$

Then $(\mathcal{Z}_\xi(\mathcal{J}),*,(0,0\mathcal{J}))isa\ \mathcal{NZ}^s$of $\mathcal{Z}(\mathcal{J})$.

Theorem 3.12: Let $\{\mathcal{A}(\mathcal{J})_\gamma : \gamma \in \mathcal{S}\}$and $\mathcal{A}(\mathcal{J})_\gamma \neq \phi$be a collection of $\mathcal{NZ}^s$ of $\mathcal{Z}(\mathcal{J})$ if

$$\bigcap_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{J})_\gamma \neq \{(0,0\mathcal{J})\} \Rightarrow \bigcap_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{J})_\gamma \ is \text{ a } \mathcal{NZ}^s\ of\ \mathcal{Z}(\mathcal{J}) .$$

Proof: since $(0,0\mathcal{J}) \in \mathcal{A}(\mathcal{J})_\gamma, \forall\, \gamma \in \mathcal{S} \Rightarrow$

$$(0,0\mathcal{J}) \in \bigcap_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{J})_\gamma \Rightarrow \bigcap_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{J})_\gamma \neq \phi$$

And the third condition was achieved for$\mathcal{A}(\mathcal{J})_\gamma, \forall\, \gamma \in \mathcal{S} \Rightarrow$

The third condition was achieved for $\bigcap_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{J})_\gamma$

$\bigcap_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma \neq \{(0,0\mathcal{I})\} \Rightarrow \exists\, (ᘔ, \text{ƀ}\mathcal{I}) \in \bigcap_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma \Rightarrow (ᘔ, \text{ƀ}\mathcal{I}) \neq (0,0\mathcal{I}) \Rightarrow$

$\{(0,0\mathcal{I})\} \subseteq \bigcap_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma$, which is a Z – algebra

Let $(ᘔ, \text{ƀ}\mathcal{I}), (\dagger, \text{Ч}\mathcal{I}) \in \bigcap_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma \Rightarrow (ᘔ, \text{ƀ}\mathcal{I}), (\dagger, \text{Ч}\mathcal{I}) \in \mathcal{A}(\mathcal{I})_\gamma, \forall\, \gamma \in \mathcal{S}$

Since $\mathcal{A}(\mathcal{I})_\gamma$ is a $\mathcal{N}\mathcal{Z}^{s}$, $\forall\, \gamma \in \mathcal{S}$ of $\mathcal{Z}(\mathcal{I})$ then

$(ᘔ, \text{ƀ}\mathcal{I}) * (\dagger, \text{Ч}\mathcal{I}) \in \mathcal{A}(\mathcal{I})_\gamma, \forall\, \gamma \in \mathcal{S}, \Rightarrow (ᘔ, \text{ƀ}\mathcal{I}) * (\dagger, \text{Ч}\mathcal{I}) \in \bigcap_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma$

hence $\bigcap_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma$ is a $\mathcal{N}\mathcal{Z}^{s}$ of $\mathcal{Z}(\mathcal{I})$.

Theorem 3.13: Let $\{\mathcal{A}(\mathcal{I})_\gamma : \gamma \in \mathcal{S}\}$ and $\mathcal{A}(\mathcal{I})_\gamma \neq \phi$ be a collection of $\mathcal{N}\text{P}\mathcal{Z}^{s} of$ $(\mathcal{Z}(\mathcal{I}), *, \circledast, (0,0\mathcal{I})$ is $\mathcal{N}\text{P}\mathcal{Z}$ if

$\bigcap_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma \neq \{(0,0\mathcal{I})\} \Rightarrow \bigcap_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma \; is$ a $\mathcal{N}\text{P}\mathcal{Z}^{s} \; of \; \mathcal{Z}(\mathcal{I})$.

Proof: it is easy as above.

Theorem 3.14: Let $\{\mathcal{A}(\mathcal{I})_\gamma : \gamma \in \mathcal{S}\}$ and $\mathcal{A}(\mathcal{I})_\gamma \neq \phi$ be a collection of $\mathcal{N}\mathcal{Z}^{s}$ of $\mathcal{Z}(\mathcal{I})$ if $\mathcal{A}(\mathcal{I})_1 \subseteq \mathcal{A}(\mathcal{I})_2 \subseteq \cdots$ then

$\bigcup_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma$ is a $\mathcal{N}\mathcal{Z}^{s}$ of $\mathcal{Z}(\mathcal{I})$.

Proof: obviously $(0,0\mathcal{I}) \in \bigcup_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma \neq \Phi \Rightarrow \exists\, (ᘔ, \text{ƀ}\mathcal{I}), (\dagger, \text{Ч}\mathcal{I}) \in \bigcup_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma$

$\Rightarrow$ Forsome $\gamma \in \mathcal{S}\, (ᘔ, \text{ƀ}\mathcal{I}), (\dagger, \text{Ч}\mathcal{I}) \in \mathcal{A}(\mathcal{I})_\gamma \; and \; (ᘔ, \text{ƀ}\mathcal{I}) * (\dagger, \text{Ч}\mathcal{I}) \in \mathcal{A}(\mathcal{I})_{\gamma\in\mathcal{S}}$

$\Rightarrow (ᘔ, \text{ƀ}\mathcal{I}) * (\dagger, \text{Ч}\mathcal{I}) \in \bigcup_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma$

Let $\mathfrak{S}(\mathcal{I})_\gamma$ be aproper sub set of $\mathcal{A}(\mathcal{I})_\gamma$, for some $\gamma \in \mathcal{S}$ which a Z- algebra,

then for any $\gamma \in \mathcal{S}, \mathfrak{S}(\mathcal{I})_\gamma \in \bigcup_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma$ then

$\bigcup_{\gamma\in\mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma\in\mathcal{S}}$ is $\mathcal{N}\mathcal{Z}^{s}$ of $\mathcal{Z}(\mathcal{I})$.

Theorem 3.15: Let $\{\mathcal{A}(\mathcal{J})_\gamma : \gamma \in \mathcal{S}\}$ and $\mathcal{A}(\mathcal{J})_\gamma \neq \phi$ be a collection of $\mathcal{N}\mathrm{P}\mathcal{Z}^{s}$ of $(\mathcal{Z}(\mathcal{J}), *, \circledast, (0,0\mathcal{J})$ is $\mathcal{N}\mathrm{P}\mathcal{Z}$ if $\mathcal{A}(\mathcal{J})_1 \subseteq \mathcal{A}(\mathcal{J})_2 \subseteq \cdots$ then

$$\bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_\gamma \text{ is } \mathcal{N}\mathrm{P}\mathcal{Z}^{s} \text{ of } \mathcal{Z}(\mathcal{J}).$$

Proof: it is easy as above.

Definition 3.16: Let $\mathcal{D}(\mathcal{J}) \neq \phi$ and $\mathcal{D}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, $(\mathcal{Z}(\mathcal{J}), *, (0,0\mathcal{J}))$ is $\mathcal{N}\mathcal{Z}$, $\mathcal{D}(\mathcal{J})$ is named a neutrosophicZ-ideal (briefly, $\mathcal{N}\mathcal{Z}^{i}$) of $\mathcal{Z}(\mathcal{J})$ if :

1) $(0,0\mathcal{J}) \in \mathcal{D}(\mathcal{J})$
2) If $(Ƨ, ђ\mathcal{J}) * (†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J})$, and $(†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow (Ƨ, ђ\mathcal{J}) \in \mathcal{D}(\mathcal{J})$

Remark 3.17: Let $\mathcal{D}(\mathcal{J})$ is a $\mathcal{N}\mathcal{Z}^{i}$ of $\mathcal{Z}(\mathcal{J})$ if

$(†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J})$ and $(Ƨ, ђ\mathcal{J}) * (†, Ч\mathcal{J}) = (0,0\mathcal{J})$ then $(Ƨ, ђ\mathcal{J}) \in \mathcal{D}(\mathcal{J})$.

Proof: let $(†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J})$ and $(Ƨ, ђ\mathcal{J}) * (†, Ч\mathcal{J}) = (0,0\mathcal{J}) \Rightarrow$

$$(†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \ \ and \ \ (0,0\mathcal{J}) \in \mathcal{D}(\mathcal{J}), (Ƨ, ђ\mathcal{J}) * (†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J})$$

Since $\mathcal{D}(\mathcal{J})$ is a $\mathcal{N}\mathcal{Z}^{i} \Rightarrow (Ƨ, ђ\mathcal{J}) \in \mathcal{D}(\mathcal{J})$.

Definition 3.18: Let $\mathcal{D}(\mathcal{J}) \neq \phi$ and $\mathcal{D}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, $(\mathcal{Z}(\mathcal{J}), *, \circledast, (0,0\mathcal{J})$ is $\mathcal{N}\mathrm{P}\mathcal{Z}$, $\mathcal{D}(\mathcal{J})$ is named a neutrosophic pseudoZ-ideal (briefly, $\mathcal{N}\mathrm{P}\mathcal{Z}^{i}$) of $\mathcal{Z}(\mathcal{J})$ if :

1) $(0,0\mathcal{J}) \in \mathcal{D}(\mathcal{J})$.
2) If $(Ƨ, ђ\mathcal{J}) * (†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J})$, and $(†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow (Ƨ, ђ\mathcal{J}) \in \mathcal{D}(\mathcal{J})$

And $(Ƨ, ђ\mathcal{J}) \circledast (†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J})$, and $(†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow (Ƨ, ђ\mathcal{J}) \in \mathcal{D}(\mathcal{J})$.

Definition 3.19: Let $\mathcal{D}(\mathcal{J}) \neq \phi$ and $\mathcal{D}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, $(\mathcal{Z}(\mathcal{J}), *, (0,0\mathcal{J}))$ is $\mathcal{N}\mathcal{Z}$, $\mathcal{D}(\mathcal{J})$ is named a neutrosophicZ_1-ideal (briefly, $\mathcal{N}\mathcal{Z}^{i1}$) of $\mathcal{Z}(\mathcal{J})$ if :

1) $(0,0\mathcal{J}) \in \mathcal{D}(\mathcal{J})$
2) If $[((Ƨ, ђ\mathcal{J}) * (ƛ, ɷ\mathcal{J})) * (Ƨ, ђ\mathcal{J})] * (†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J})$, and $(†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow$ $(Ƨ, ђ\mathcal{J}) \in \mathcal{D}(\mathcal{J}), \forall (Ƨ, ђ\mathcal{J}), (ƛ, ɷ\mathcal{J}), (†, Ч\mathcal{J}) \in \mathcal{Z}(\mathcal{J})$

Definition 3.20: : Let $\mathcal{D}(\mathcal{J}) \neq \phi$ and $\mathcal{D}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, $(\mathcal{Z}(\mathcal{J}), *, \circledast, (0,0\mathcal{J})$ is $\mathcal{N}\mathrm{P}\mathcal{Z}$, $\mathcal{D}(\mathcal{J})$ is named a neutrosophic pseudo Z_1-ideal (briefly, $\mathcal{N}\mathrm{P}\mathcal{Z}^{i1}$) of $\mathcal{Z}(\mathcal{J})$ if :

1) $(0,0\mathcal{J}) \in \mathcal{D}(\mathcal{J})$
2) $[((Ƨ, ђ\mathcal{J}) * (ƛ, ɷ\mathcal{J})) * (Ƨ, ђ\mathcal{J})] * (†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J})$, and $(†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow$ $(Ƨ, ђ\mathcal{J}) \in \mathcal{D}(\mathcal{J}), \forall (Ƨ, ђ\mathcal{J}), (ƛ, ɷ\mathcal{J}), (†, Ч\mathcal{J}) \in \mathcal{Z}(\mathcal{J})$

And $\left[\left((Ƨ, ђ\mathcal{J}) \circledast (ƛ, ɷ\mathcal{J})\right) \circledast (Ƨ, ђ\mathcal{J})\right] \circledast (†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J})$, and $(†, Ч\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow$ $(Ƨ, ђ\mathcal{J}) \in \mathcal{D}(\mathcal{J}), \forall (Ƨ, ђ\mathcal{J}), (ƛ, ɷ\mathcal{J}), (†, Ч\mathcal{J}) \in \mathcal{Z}(\mathcal{J})$

Definition 3.21: Let$\mathcal{D}(\mathcal{I}) \neq \phi$ and $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$,$(\mathcal{Z}(\mathcal{I}),*,(0,0\mathcal{I}))$ is $\mathcal{NZ}$,$\mathcal{D}(\mathcal{I})$ is named a neutrosophicZ_2-ideal (briefly, $\mathcal{NZ}^{i2}$) of $\mathcal{Z}(\mathcal{I})$ if :

1) $(0,0\mathcal{I}) \in \mathcal{D}(\mathcal{I})$
2) If $[(\text{ƨ},\text{ḅ}\mathcal{I}) * (\text{ϡ},\text{ω}\mathcal{I})] * [(\text{ƨ},\text{ḅ}\mathcal{I}) * (\text{ϯ},\text{Ч}\mathcal{I})] \in \mathcal{D}(\mathcal{I})$, and $(\text{ϯ},\text{Ч}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow$ $(\text{ƨ},\text{ḅ}\mathcal{I}) \in \mathcal{D}(\mathcal{I}), \forall (\text{ƨ},\text{ḅ}\mathcal{I}),(\text{ϡ},\text{ω}\mathcal{I}),(\text{ϯ},\text{Ч}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$.

Definition 3.22: Let$\mathcal{D}(\mathcal{I}) \neq \phi$ and $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$,$(\mathcal{Z}(\mathcal{I}),*,\circledast,(0,0\mathcal{I})$ is $\mathcal{N}\mathrm{P}\mathcal{Z}$,$\mathcal{D}(\mathcal{I})$ is named a neutrosophicpseudo Z_2-ideal (briefly, $\mathcal{N}\mathrm{P}\mathcal{Z}^{i2}$) of $\mathcal{Z}(\mathcal{I})$ if :

1) $(0,0\mathcal{I}) \in \mathcal{D}(\mathcal{I})$
2) If $[(\text{ƨ},\text{ḅ}\mathcal{I}) * (\text{ϡ},\text{ω}\mathcal{I})] * [(\text{ƨ},\text{ḅ}\mathcal{I}) * (\text{ϯ},\text{Ч}\mathcal{I})] \in \mathcal{D}(\mathcal{I})$, and $(\text{ϯ},\text{Ч}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow$ $(\text{ƨ},\text{ḅ}\mathcal{I}) \in \mathcal{D}(\mathcal{I}), \forall (\text{ƨ},\text{ḅ}\mathcal{I}),(\text{ϡ},\text{ω}\mathcal{I}),(\text{ϯ},\text{Ч}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$

And$[(\text{ƨ},\text{ḅ}\mathcal{I}) \circledast (\text{ϡ},\text{ω}\mathcal{I})] \circledast [(\text{ƨ},\text{ḅ}\mathcal{I}) \circledast (\text{ϯ},\text{Ч}\mathcal{I})] \in \mathcal{D}(\mathcal{I})$, and $(\text{ϯ},\text{Ч}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow$ $(\text{ƨ},\text{ḅ}\mathcal{I}) \in \mathcal{D}(\mathcal{I}), \forall (\text{ƨ},\text{ḅ}\mathcal{I}),(\text{ϡ},\text{ω}\mathcal{I}),(\text{ϯ},\text{Ч}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$.

Definition 3.23: Let$\mathcal{D}(\mathcal{I}) \neq \phi$ and $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$,$(\mathcal{Z}(\mathcal{I}),*,(0,0\mathcal{I}))$ is $\mathcal{NZ}$,$\mathcal{D}(\mathcal{I})$ is named a neutrosophicZ_q-ideal (briefly, $\mathcal{NZ}^{iq}$) of $\mathcal{Z}(\mathcal{I})$ if :

1) $(0,0\mathcal{I}) \in \mathcal{D}(\mathcal{I})$
2) If $[(\text{ƨ},\text{ḅ}\mathcal{I}) * (\text{ϡ},\text{ω}\mathcal{I})] * [(\text{ϯ},\text{Ч}\mathcal{I}) * (\text{ϡ},\text{ω}\mathcal{I})] \in \mathcal{D}(\mathcal{I})$, and $(\text{ϯ},\text{Ч}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow$ $(\text{ƨ},\text{ḅ}\mathcal{I}) \in \mathcal{D}(\mathcal{I}), \forall (\text{ƨ},\text{ḅ}\mathcal{I}),(\text{ϡ},\text{ω}\mathcal{I}),(\text{ϯ},\text{Ч}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$

Definition 3.24: Let$\mathcal{D}(\mathcal{I}) \neq \phi$ and $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$,$(\mathcal{Z}(\mathcal{I}),*,\circledast,(0,0\mathcal{I})$ is $\mathcal{N}\mathrm{P}\mathcal{Z}$,$\mathcal{D}(\mathcal{I})$ is named a neutrosophicpseudo Z_q ideal (briefly, $\mathcal{N}\mathrm{P}\mathcal{Z}^{iq}$) of $\mathcal{Z}(\mathcal{I})$ if :

1) $(0,0\mathcal{I}) \in \mathcal{D}(\mathcal{I})$
2) If $[(\text{ƨ},\text{ḅ}\mathcal{I}) * (\text{ϡ},\text{ω}\mathcal{I})] * [(\text{ϯ},\text{Ч}\mathcal{I}) * (\text{ϡ},\text{ω}\mathcal{I})] \in \mathcal{D}(\mathcal{I})$, and $(\text{ϯ},\text{Ч}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow$ $(\text{ƨ},\text{ḅ}\mathcal{I}) \in \mathcal{D}(\mathcal{I}), \forall (\text{ƨ},\text{ḅ}\mathcal{I}),(\text{ϡ},\text{ω}\mathcal{I}),(\text{ϯ},\text{Ч}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$

And$[(\text{ƨ},\text{ḅ}\mathcal{I}) \circledast (\text{ϡ},\text{ω}\mathcal{I})] \circledast [(\text{ϯ},\text{Ч}\mathcal{I}) \circledast (\text{ϡ},\text{ω}\mathcal{I})] \in \mathcal{D}(\mathcal{I})$, and $(\text{ϯ},\text{Ч}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow$ $(\text{ƨ},\text{ḅ}\mathcal{I}) \in \mathcal{D}(\mathcal{I}), \forall (\text{ƨ},\text{ḅ}\mathcal{I}),(\text{ϡ},\text{ω}\mathcal{I}),(\text{ϯ},\text{Ч}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$

Definition 3.25: Let$\mathcal{D}_\xi(\mathcal{I}) \neq \phi$ and $\mathcal{D}_\xi(\mathcal{I}) \subseteq \mathcal{Z}_\xi(\mathcal{I})$,$\mathcal{D}_\xi(\mathcal{I})$is named a neutrosophic Z-ideal (briefly, $\mathcal{NZ}^{\xi i}$) of $\mathcal{Z}_\xi(\mathrm{I})$ if :

1) $(0,0\mathcal{I}) \in \mathcal{D}_\xi(\mathcal{I})$
2) If $(\text{ƨ},\text{Ƨ}\mathcal{I}) * [(\text{ϯ},\text{ϯ}\mathcal{I}) * (\text{ϡ},\text{ϡ}\mathcal{I})] \in \mathcal{D}_\xi(\mathcal{I})$, and $(\text{ϯ},\text{ϯ}\mathcal{I}) \in \mathcal{D}_\xi(\mathcal{I})$
$\Rightarrow (\text{ƨ},\text{Ƨ}\mathcal{I}) * (\text{ϡ},\text{ϡ}\mathcal{I}) \in \mathcal{D}_\xi(\mathcal{I}), \quad \forall (\text{ƨ},\text{Ƨ}\mathcal{I}),(\text{ϯ},\text{ϯ}\mathcal{I}),(\text{ϡ},\text{ϡ}\mathcal{I})\mathcal{D}_\xi(\mathcal{I})$

Theorem 3.26: Every $\mathcal{NZ}^{\xi i}$ of $\mathcal{X}_\xi(\mathcal{I})$ is a $\mathcal{NZ}^{i}$of $\mathcal{X}_\xi(\mathcal{I})$.

Proof: suppose that $(\text{ƨ},\text{Ƨ}\mathcal{I}) = (0,0\mathcal{I})$ in 2 $\Rightarrow$ it's proofed. .

Definition 3.27: Let$\mathcal{D}(\mathcal{I}) \neq \phi$ and $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$,$(\mathcal{Z}(\mathcal{I}),*,(0,0\mathcal{I}))$ is $\mathcal{NZ}$,$\mathcal{D}(\mathcal{I})$ is named a neutrosophicZ-filter(briefly, $\mathcal{NZ}^{f}$) of $\mathcal{Z}(\mathcal{I})$if :

1) $(0,0\mathcal{I}) \notin \mathcal{D}(\mathcal{I})$
2) $\forall (Ƨ, ƅ\mathcal{I}), (ϯ, Ч\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ and $(Ƨ, ƅ\mathcal{I}) \neq (ϯ, Ч\mathcal{I}) \Rightarrow$
$$(Ƨ, ƅ\mathcal{I}) ⧖ (ϯ, Ч\mathcal{I}) = (Ƨ, ƅ\mathcal{I}) * [(Ƨ, ƅ\mathcal{I}) * (ϯ, Ч\mathcal{I})] \in \mathcal{D}(\mathcal{I})$$

Definition 3.28: Let $\mathcal{D}(\mathcal{I}) \neq \phi$ and $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$, $(\mathcal{Z}(\mathcal{I}), *, \circledast, (0,0\mathcal{I}))$ is $\mathcal{N}\mathrm{P}\mathcal{Z}$, $\mathcal{D}(\mathcal{I})$ is named a neutrosophic pseudo Z-filter (briefly, $\mathcal{N}\mathrm{P}\mathcal{Z}^{f}$) of $\mathcal{Z}(\mathcal{I})$ if :

1) $(0,0\mathcal{I}) \notin \mathcal{D}(\mathcal{I})$
2) $\forall (Ƨ, ƅ\mathcal{I}), (ϯ, Ч\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ and $(Ƨ, ƅ\mathcal{I}) \neq (ϯ, Ч\mathcal{I}) \Rightarrow$
$$(Ƨ, ƅ\mathcal{I}) ⧖ (ϯ, Ч\mathcal{I}) = (Ƨ, ƅ\mathcal{I}) * [(Ƨ, ƅ\mathcal{I}) * (ϯ, Ч\mathcal{I})] \in \mathcal{D}(\mathcal{I})$$

And $\forall (Ƨ, ƅ\mathcal{I}), (ϯ, Ч\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ and $(Ƨ, ƅ\mathcal{I}) \neq (ϯ, Ч\mathcal{I}) \Rightarrow$

$$(Ƨ, ƅ\mathcal{I}) ⧖ (ϯ, Ч\mathcal{I}) = (Ƨ, ƅ\mathcal{I}) \circledast [(Ƨ, ƅ\mathcal{I}) \circledast (ϯ, Ч\mathcal{I})] \in \mathcal{D}(\mathcal{I})$$

Definition 3.29: If $(\mathcal{Z}(\mathcal{I}), *, (0,0\mathcal{I}))$ & $\left(\mathcal{Z}(\acute{\mathcal{I}}), \acute{*}, (\acute{0}, \acute{0}\mathcal{I})\right)$ be two $\mathcal{N}\mathcal{Z}$, a mapping $f: \mathcal{Z}(\mathcal{I}) \rightarrow \mathcal{Z}(\acute{\mathcal{I}})$ is named a neutrosophic Z-homomorphism(briefly, $\mathcal{N}\mathcal{Z}^{\hbar}$)if satisfied

1) $f[(Ƨ, ƅ\mathcal{I}) * (ϯ, Ч\mathcal{I})] = f(Ƨ, ƅ\mathcal{I}) \acute{*} f(ϯ, Ч\mathcal{I}), \forall (Ƨ, ƅ\mathcal{I}), (ϯ, Ч\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$
2) $f(0,0\mathcal{I}) = (\acute{0}, \acute{0}\mathcal{I})$
3) If f is 1-1$\Rightarrow f$ is nameda neutrosophic Z-monomorphism.
4) If f is onto$\Rightarrow f$ isnameda neutrosophic Z- epimorphism.
5) If f is 1-1 and onto$\Rightarrow f$ is named a neutrosophic Z-isomorphism.

Definition 3.30: If $(\mathcal{Z}(\mathcal{I}), *, \circledast, (0,0\mathcal{I}))$ & $\left(\mathcal{Z}(\acute{\mathcal{I}}), \acute{*}, \acute{\circledast}, (\acute{0}, \acute{0}\mathcal{I})\right)$ be two $\mathcal{N}\mathrm{P}\mathcal{Z}$, a mapping $f: \mathcal{Z}(\mathcal{I}) \rightarrow \mathcal{Z}(\acute{\mathcal{I}})$ is named a neutrosophic pseudo Z- homomorphism (briefly, $\mathcal{N}\mathrm{P}\mathcal{Z}^{\hbar}$) if satisfied

1) $f[(Ƨ, ƅ\mathcal{I}) * (ϯ, Ч\mathcal{I})] = f(Ƨ, ƅ\mathcal{I}) \acute{*} f(ϯ, Ч\mathcal{I}), \forall (Ƨ, ƅ\mathcal{I}), (ϯ, Ч\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$
2) $f[(Ƨ, ƅ\mathcal{I}) \circledast (ϯ, Ч\mathcal{I})] = f(Ƨ, ƅ\mathcal{I}) \acute{\circledast} f(ϯ, Ч\mathcal{I}), \forall (Ƨ, ƅ\mathcal{I}), (ϯ, Ч\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$
3) $f(0,0\mathcal{I}) = (\acute{0}, \acute{0}\mathcal{I})$
4) If f is 1-1 $\Rightarrow f$ is named"a neutrosophic pseudo Z- monomorphism".
5) If f is onto $\Rightarrow f$ isnamed"a neutrosophic pseudo Z- epimorphism".
6) If f is 1-1 and onto $\Rightarrow f$ is named a neutrosophic pseudo Z-isomorphism.

Theorem 3.31: Let $\mathcal{Z}(\mathcal{I})$ & $\mathcal{Z}(\acute{\mathcal{I}})$ be two $\mathcal{N}\mathcal{Z}$, $f: \mathcal{Z}(\mathcal{I}) \rightarrow \mathcal{Z}(\acute{\mathcal{I}})$ be a neutrosophic Z-epimorphism.If $\mathcal{D}(\mathcal{I})$ is a $\mathcal{N}\mathcal{Z}^{f}$ of $\mathcal{Z}(\mathcal{I}) \Rightarrow f(\mathcal{D}(\mathcal{I}))$ is a $\mathcal{N}\mathcal{Z}^{f}$ of $\mathcal{Z}(\acute{\mathcal{I}})$.

Proof: let $(Ƨ, ƅ\mathcal{I}), (ϯ, Ч\mathcal{I}) \in f(\mathcal{D}(\mathcal{I})) \Rightarrow$

$(Ƨ, ƅ\mathcal{I}) = f(ঽ, ɷ\mathcal{I})$, $(ϯ, Ч\mathcal{I}) = f(ϙ, ϵ\mathcal{I})$ where $(ঽ, ɷ\mathcal{I}), (ϙ, ϵ\mathcal{I}) \in \mathcal{D}(\mathcal{I})$

Since $\mathcal{D}(\mathcal{I})$ is a $\mathcal{N}\mathcal{Z}^{f}$ of $\mathcal{Z}(\mathcal{I}), \Rightarrow$

$(ঽ, ɷ\mathcal{I}) ⧖ (ϙ, ϵ\mathcal{I}) = (ঽ, ɷ\mathcal{I}) * [(ঽ, ɷ\mathcal{I}) * (ϙ, ϵ\mathcal{I})] \in \mathcal{D}(\mathcal{I})$

Also $f\left((ঽ, ɷ\mathcal{I}) ⧖ (ʒ, ŋ\mathcal{I})\right) \in f(\mathcal{D}(\mathcal{I}))$

$(ƻ, ƅ\mathcal{I}) ⴳ (†, Ч\mathcal{I}) = (ƻ, ƅ\mathcal{I}) * \left((ƻ, ƅ\mathcal{I}) * (†, Ч\mathcal{I})\right)$

$= f\ (ঌ, ယ\mathcal{I}) \acute{*} \left(f\ (ঌ, ယ\mathcal{I}) \acute{*} f\ (ϙ, ϵ\mathcal{I})\right)$

$= f\ \left[(ঌ, ယ\mathcal{I}) * \left((ঌ, ယ\mathcal{I}) * \ (ϙ, ϵ\mathcal{I})\right)\right]$

$= \ f\ [(ঌ, ယ\mathcal{I}) \Delta\ (ϙ, ϵ\mathcal{I})]$

$(ƻ, ƅ\mathcal{I}) ⴳ (†, Ч\mathcal{I}) \in f(\mathcal{D}(\mathcal{I})) \ \Rightarrow$

$f(\mathcal{D}(\mathcal{I}))$is a $\mathcal{NZ}^{f}$ of $\mathcal{Z}(\acute{\mathcal{I}})$.

Theorem 3.32: Let$\mathcal{Z}(\mathcal{I})$&$\mathcal{Z}(\acute{\mathcal{I}})$ be two$\mathcal{N}\mathsf{P}\mathcal{Z}$, $f: \mathcal{Z}(\mathcal{I}) \to \mathcal{Z}(\acute{\mathcal{I}})$ be a neutrosophic pseudo Z- epimorphism.If $\mathcal{D}(\mathcal{I})$is a $\mathcal{N}\mathsf{P}\mathcal{Z}^{f}$ of $\mathcal{Z}(\mathcal{I}) \Rightarrow f(\mathcal{D}(\mathcal{I}))$ is a $\mathcal{N}\mathsf{P}\mathcal{Z}^{f}$ of $\mathcal{Z}(\acute{\mathcal{I}})$.

Proof: it is easy as above.

Definition 3.33: Let$f: \mathcal{Z}(\mathcal{I}) \to \mathcal{Z}(\acute{\mathcal{I}})$ be a $\mathcal{NZ}^{\hbar}$then
$\ker(f) = \left\{(ƻ, ƅ\mathcal{I}) \in \ \mathcal{Z}(\mathcal{I}): f(ƻ, ƅ\mathcal{I}) = \left(\acute{0}, \acute{0}\mathcal{I}\right)\right\}$ is named the kernel of f.

Definition 3.34: Let$f: \mathcal{Z}(\mathcal{I}) \to \mathcal{Z}(\acute{\mathcal{I}})$ be a $\mathcal{N}\mathsf{P}\mathcal{Z}^{\hbar}$then

$\ker(f) = \left\{(ƻ, ƅ\mathcal{I}) \in \ \mathcal{Z}(\mathcal{I}): f(ƻ, ƅ\mathcal{I}) = \left(\acute{0}, \acute{0}\mathcal{I}\right)\right\}$is named the kernel of f .

Remark 3.35: (1)Let $f: \mathcal{Z}(\mathcal{I}) \to \mathcal{Z}(\acute{\mathcal{I}})$ is a $\mathcal{NZ}^{\hbar}$,then $\ker(f)$ is not a $\mathcal{NZ}^{f}$ of $\mathcal{Z}(\mathcal{I})$.

(2) $\mathcal{NZ}^{f}$is not $\mathcal{NZ}^{i}$ and conversely .

(3) $\mathcal{NZ}^{f}$is not $\mathcal{NZ}^{s}$ and conversely .

Remark 3.36: (1) Let $f: \mathcal{Z}(\mathcal{I}) \to \mathcal{Z}(\acute{\mathcal{I}})$ is a $\mathcal{N}\mathsf{P}\mathcal{Z}^{\hbar}$, then $\ker(f)$ is not a $\mathcal{N}\mathsf{P}\mathcal{Z}^{f}$ of $\mathcal{Z}(\mathcal{I})$.

(2) $\mathcal{N}\mathsf{P}\mathcal{Z}^{f}$is not $\mathcal{N}\mathsf{P}\mathcal{Z}^{i}$ and conversely .

(3) $\mathcal{N}\mathsf{P}\mathcal{Z}^{f}$is not $\mathcal{N}\mathsf{P}\mathcal{Z}^{s}$ and conversely .

Theorem 3.37: Let$f: \mathcal{Z}(\mathcal{I}) \to \mathcal{Z}(\acute{\mathcal{I}})$ be a $\mathcal{NZ}^{\hbar}$then

1) If the identity of $\mathcal{Z}(\mathcal{I})$ is $(0,0\mathcal{I}) \Rightarrow$ the identity of$\mathcal{Z}(\acute{\mathcal{I}})isf(0,0\mathcal{I})$.
2) If $\mathcal{U}$ is a $\mathcal{NZ}^{s}$of$\mathcal{Z}(\mathcal{I})$, then $f(\mathcal{U})$ isa$\mathcal{NZ}^{s}$of $\mathcal{Z}(\acute{\mathcal{I}})$.
3) If $\mathcal{U}$ is a $\mathcal{NZ}^{s}$of $\mathcal{Z}(\acute{\mathcal{I}})$, then $f^{-1}(\mathcal{U})$ isa $\mathcal{NZ}^{s}$of $\mathcal{Z}(\mathcal{I})$.

Proof: it's clear.

Theorem 3.38: Let$f: \mathcal{Z}(\mathcal{I}) \to \mathcal{Z}(\acute{\mathcal{I}})$ be a $\mathcal{N}\mathsf{P}\mathcal{Z}^{\hbar}$then

1) If the identity of $\mathcal{Z}(\mathcal{I})$ is $(0,0\mathcal{I}) \Rightarrow$ the identity of $\mathcal{Z}(\acute{\mathcal{I}})$is $f(0,0\mathcal{I})$.
2) If $\mathcal{U}$ is a $\mathcal{N}\mathsf{P}\mathcal{Z}^{s}$ of$\mathcal{Z}(\mathcal{I})$, then $f(\mathcal{U})$ isa$\mathcal{N}\mathsf{P}\mathcal{Z}^{s}$of $\mathcal{Z}(\acute{\mathcal{I}})$.
3) If $\mathcal{U}$ is a $\mathcal{N}\mathsf{P}\mathcal{Z}^{s}$of $\mathcal{Z}(\acute{\mathcal{I}})$, then $f^{-1}(\mathcal{U})$ isa $\mathcal{N}\mathsf{P}\mathcal{Z}^{s}$of $\mathcal{Z}(\mathcal{I})$.

Proof: it's clear.

Theorem 3.39: Let $f: \mathcal{Z}(\mathcal{I}) \to \acute{\mathcal{Z}}(\mathcal{I})$ is a $\mathcal{NZ}^{\hbar}$ then f is a neutrosophic Z- monomorphism $\Leftrightarrow \ker(f) = \{(0,0I)\}$

Proof: it's clear.

Theorem 3.40: Let $f: \mathcal{Z}(\mathcal{I}) \to \acute{\mathcal{Z}}(\mathcal{I})$ is a $\mathcal{N}\mathrm{P}\mathcal{Z}^{\hbar}$ then f is a neutrosophic Z-monomorphism $\Leftrightarrow \ker(f) = \{(0,0I)\}$

Proof: it's clear.

Theorem 3.41: Let $f: \mathcal{Z}(\mathcal{I}) \to \acute{\mathcal{Z}}(\mathcal{I})$ is a $\mathcal{NZ}^{\hbar}$ then $\ker(f)$ is a $\mathcal{NZ}^{i}$ of $\mathcal{Z}(\mathcal{I})$.

Proof: $f(0,0\mathcal{I}) = (\acute{0},\acute{0}\mathcal{I}) \Rightarrow (0,0\mathcal{I}) \in \ker(f)$

Let $(Ϩ, ƀ\mathcal{I}) * [(ϯ, Ч\mathcal{I}) * (ϡ, ω\mathcal{I})] \in \ker(f)$ and $(ϯ, Ч\mathcal{I}) \in \ker(f) \Rightarrow$

$f\big((Ϩ, ƀ\mathcal{I}) * [(ϯ, Ч\mathcal{I}) * (ϡ, ω\mathcal{I})]\big) = (\acute{0},\acute{0}\mathcal{I})$ and $f(ϯ, Ч\mathcal{I}) = (\acute{0},\acute{0}\mathcal{I})$

$$\begin{aligned}(\acute{0},\acute{0}\mathcal{I}) &= f\big((Ϩ, ƀ\mathcal{I}) * [(ϯ, Ч\mathcal{I}) * (ϡ, ω\mathcal{I})]\big)\\ &= f(Ϩ, ƀ\mathcal{I}) \acute{*} [f(ϯ, Ч\mathcal{I}) \acute{*} f(ϡ, ω\mathcal{I})]\\ &= f(Ϩ, ƀ\mathcal{I}) \acute{*} \big[(\acute{0},\acute{0}\mathcal{I}) \acute{*} f(ϡ, ω\mathcal{I})\big]\\ &= f(Ϩ, ƀ\mathcal{I}) \acute{*} f(ϡ, ω\mathcal{I})\\ &= f\big((Ϩ, ƀ\mathcal{I}) * (ϡ, ω\mathcal{I})\big)\end{aligned}$$

We get $\big((Ϩ, ƀ\mathcal{I}) * (ϡ, ω\mathcal{I})\big) \in \ker(f)$.then $\ker(f)$ is a a$\mathcal{NZ}^{i}$ of $\mathcal{Z}(\mathcal{I})$.

Theorem 3.42: Let $f: \mathcal{Z}(\mathcal{I}) \to \acute{\mathcal{Z}}(\mathcal{I})$ is a $\mathcal{N}\mathrm{P}\mathcal{Z}^{\hbar}$ then $\ker(f)$ is a $\mathcal{N}\mathrm{P}\mathcal{Z}^{i}$ of $\mathcal{Z}(\mathcal{I})$.

Proof: it is easy as above.

CONCLUSION:

We discussed the idea of a neutrosophic Z-algebra andneutrosophic$\mathrm{pseudo}Z-\mathrm{algebra}$ looked into some of its properties,and the concept of neutrosophicZ-ideal,neutrosophicZ-sub algebra,neutrosophicZ-filter and neutrosophicZ-homomorphism are studied and a few properties are obtained.

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