On the Effect ofSeasonal Averages and Standard Deviations on Buys-Ballot Estimates of Time Series Components in the Presence of Missing Values

Abstract: In this study, weconsider theeffect of seasonal averages and seasonal standard deviations on Buys-Ballot estimates of time series components inthe presence of missing values. The emphasis is to compare seasonal averages with seasonal standard deviations in the presence and absence of missing values using real life example.The methods of mean and regression imputations are adopted for estimating missing values in time series data. Result of this analysis shows that, the differences between Seasonal averages and seasonal standard deviations with and without missing values listed in Tables7 and 9, have insignificant effect on the Buys-Ballot estimates of time series components.

Keywords:Missing Data, Additive Model, Mean Imputation, Regression Imputation, Standard Deviation,Quadratic Trend.

1 Introduction

The analysis of time series, a problem usually encountered in data collection is a missing values. Missing values may be virtually impossible to get, either because of time or cost constraint. To obtain estimates of these values, there are many options available to the researcher. One of them is to replace them by mean of the data. The missing values may be replaced with naïve forecast or with the average of the last two known observations that bound the missing value Almed [1].

Brockwell and Davis [2] discussed the option that missing data at the beginning or end of the series are simply ignored while intermediate missing data are seen as problems in the input time series. Therefore, they observed that, interpolates values using interpolation algorithm linear, polynomial, smoothing, spline and filering.

Cheema [3]gave comparison of different methods of handling missing data. They include mean imputation, regression imputation, maximum likelihood imputation, multiple imputation and listwise deletion.

Iwuezee*t.al* [4] proposed three methods in estimating of missing values. The methods are, Row Mean Imputation (RMI), Column Mean Imputation (CMI) and Decomposition Without the Missing Values (DWMV). They added that, Decomposition Without the Missing Values (DWMV) yielded best in terms of accuracy measures estimates of the missing values when compared with other existing methods. Therefore, they recommended that, the Decomposition Without the Missing Values (DWMV) method be used in estimating missing values in time series analysis when one observation is missing a time in the Buys-Ballot table.

The rationale for this study is actually to contribute knowledge to existing literature in estimation of missing values by providing analyst the functional relationship between seasonal averages and seasonal standard deviation and the effect on Buys-Ballot estimates of time series components.

2. Materials and Methods

The methods used in this study are 1) the Buys-Ballot Procedure (BBP), 2) the Mean (MI) Imputation 3) the Regression Imputation (RI). The study series arearranged in a Buys-Ballot Table with m periods and s seasons. For details of Buys-Ballot table, seeIwueze*et.al* [4], Akpanta and Iwueze [5],Nwogu*et.al* [6], Dozie*et.al* [7], Dozie and Ihekuna [8],Dozie and Ibebuogu [9], Dozie and Ijeomah [10], Dozie and Nwanya [11], Dozie [12]

2.1Mean Imputation (MI)

Mean imputations is given by

$$MI = \hat{X}_{(i-1)s+j} = \frac{1}{(i-1)s+j-1}\left[X_1 + X_2 + X_3 + \ldots + X_{(i-1)s+j}\right] = \frac{1}{n}\sum_{i=1}^{n} X_i \quad (1)$$

Comment [Ma1]: Author need to explain what is the meaning of mathematical symbols in each equation. For example, Equation (1) here, the X cap represents for what? And n is stands for size sample or something else?Please state clearly each of the symbols.

2.2Regression Imputation (RI)

Regression imputation is given by

$$\hat{X}_{(i-1)s+j} = \hat{a} + \hat{b}[(i-1)s+j] + \hat{c}[(i-1)s+j]^2 \qquad (2)$$

2.3Estimation of Trend Parameters and Seasonal Indices

Iwueze and Nwogu [13] provided estimation for trend and seasonal indices shown in equations (3), (4), (5), (6), and (7) below:

Comment [Ma2]: Which equation represents for trend and seasonal?

$$\hat{a} = a' + \left(\frac{s-1}{2}\right)\hat{b} - \left(\frac{(s-1)(2s-1)}{6}\right)\hat{c} \qquad (3)$$

$$\hat{b} = \frac{b'}{s} + \hat{c}(s-1) \qquad (4)$$

$$\hat{c} = \frac{c'}{s^2} \qquad (5)$$

$$\hat{S}_j = \bar{X}_{.j} - d_j \qquad (6)$$

$$d_j = \hat{a} + \frac{\hat{b}}{2}(n-s) + \frac{\hat{c}(n-s)(2n-s)}{6} + \hat{b} + \hat{c}(n-s)j + \hat{c}\, j^2 \qquad (7)$$

2.4Estimation of Missing data in the Transformed and Untransformed Time Series

The transformed estimated missing data is given by;

$$\hat{X}_{ij} = \hat{a} + \hat{b}[(i-1)s+j] + \hat{c}[(i-1)s+j]^2 + \hat{S}_j \qquad (8)$$

The estimate of the missing data in the untransformed series.

$$Untransformed \quad \hat{X}_{ij} = e^{\hat{a} + \hat{b}[(i-1)s+j] + \hat{c}[(i-1)s+j]^2 + \hat{S}_j} \qquad (9)$$

Results and discussion

3. Real Life Example:

This section contains real life dataexample to illustrate the methods of mean and regression imputations in estimating trend parameters and seasonal indices in the presence and absence of missing values. One hundred and eight (108) registered road accidents are considered from January to December 2013 to 2021 in which three (3) registered accidents are not accounted for.The time plots of actual and transformed series with missing data are given in figure 3.1,3.2,3.3and 3.4. The amplitude of figure 3.1 appears to have increased in

Comment [Ma3]: Where did the author got the sample data of road accidents? Please state it. If it comes from open source, please state the website here.

the later year indicating that the variance is not constant, suggesting that the data requires transformation to stabilize the variance. The natural logarithm of the periodic and standard deviation are given in Table 1. The data is transformed by using the natural logarithm of the one hundred and seventeen by the method of Akpantaand Iwueze[5].

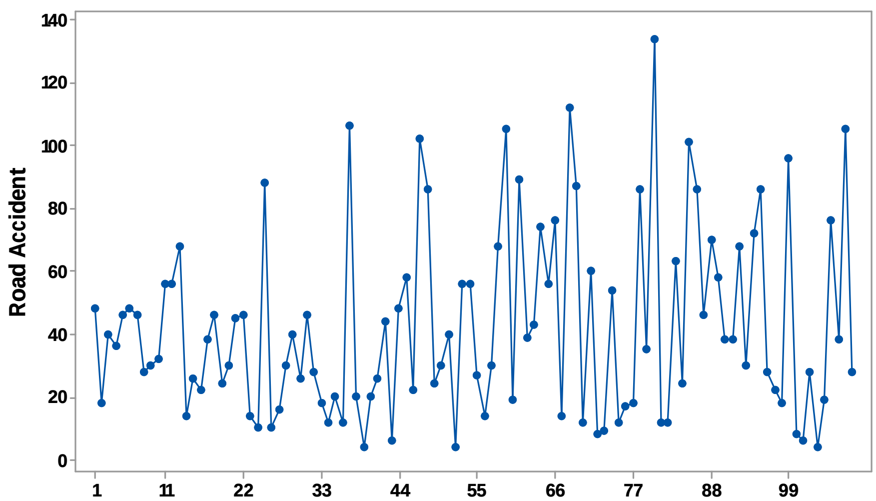

Fig 3.1: Time plot of actual series with missing observations

Table 1: Natural Logarithm of Periodic Averages and Standard DeviationsWith Missing Values

$\bar{X}_{i.}$	$Log_e \bar{X}_{i.}$	$\hat{\sigma}_{i.}$	$Log_e \hat{\sigma}_{i.}$
40.33	3.70	11.75	2.46
31.92	3.46	17.13	2.84
28.83	3.36	21.77	3.08
45.20	3.81	35.90	3.58
39.42	3.67	27.89	3.33
55.83	4.02	33.56	3.51
39.70	3.68	38.50	3.65
60.08	4.10	24.18	3.19
37.30	3.62	35.10	3.56

3.1 Estimates with Missing Observations

The estimates with missing observations is given as;

$$\bar{X}_i = 3.437 - 0.0107t + 0.0401t^2 \tag{10}$$

Where, $a^| = 3.437,\ b^| = -0.0107\ c^| = 0.0401$

Using (3),(4) and (5) we obtain

$\hat{a} = 3.5563, \hat{b} = 0.024, \hat{c} = 0.0003,$

$d_j = \bar{X}_{.j} - \hat{S}_{.j} = \bar{X}_{.j} - 5.7163 - 0.0003_{j^2}$

Table 2: Estimates of Seasonal Indices WithMissing Values

j	$\bar{X}_{.j}$	$Adj\ \hat{S}_j$
1	3.8670	1.4410
2	3.2470	-0.2443
3	3.2700	-0.2983
4	3.1030	-0.0366
5	3.4670	-0.0798
6	3.8360	-0.0555
7	3.0160	-0.3158
8	3.7060	-0.3177
9	3.5830	-0.1922
10	3.3300	-0.5083
11	4.0320	-0.3951
12	3.1260	0.9294

.3.2 Estimates without Missing Observations

Here, estimates without missing observations is given as;

$$\bar{X}_i = 3.437 - 0.0108t + 0.0404t^2 \tag{11}$$

Where, $a^| = 3.437,\ b^| = -0.0108\ c^| = 0.0404$

Using (3), (4) and (5) we obtain

$\hat{a} = 3.5563, \hat{b} = 0.024, \hat{c} = 0.0003,$

$d_j = \bar{X}_{.j} - \hat{S}_{.j} = \bar{X}_{.j} - 5.7163 - 0.0003_{j^2}$

Table 3: Estimates of Seasonal Indices WithoutMissing Values

j	$\bar{X}_{.j}$	$Adj\ \hat{S}_j$
1	3.8670	1.4410
2	3.2470	-0.2443
3	3.2700	-0.2983
4	3.1030	-0.0366
5	3.4670	-0.0798
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8	3.7060	-0.3177
9	3.5830	-0.1922
10	3.3300	-0.5083
11	4.0320	-0.3951
12	3.1260	0.9294

Table 4: Transformed Series of Natural Logarithm of Periodic Averages and Periodic Standard DeviationsWith Missing Values

$\bar{X}_{i.}$	$Log_e\ \bar{X}_{i.}$	$\hat{\sigma}_{i.}$	$Log_e\ \hat{\sigma}_{i.}$
3.6510	1.2950	0.3335	-1.0981
3.3190	1.1997	0.5810	-0.5430
3.1600	1.1506	0.6360	-0.4526
3.4150	1.2282	1.0490	0.0478
3.4050	1.2252	0.8530	-0.1589
3.7350	1.3231	0.8770	-0.1312
3.2990	1.1936	0.8920	-0.1143
4.0140	1.3898	0.4340	-0.8347
3.1700	1.1537	1.0520	0.0507

Table 5: Transformed Series of Natural Logarithm of Seasonal Averages and Seasonal Standard DeviationsWith Missing Values

$\bar{X}_{.j}$	$Log_e \bar{X}_{.j}$	$\hat{\sigma}_{.j}$	$Log_e \hat{\sigma}_{.j}$
3.8670	1.3525	0.8620	-0.1485
3.2470	1.7777	0.6880	-0.3740
3.2700	1.1848	0.9350	-0.0672
3.1030	1.1324	0.9460	-0.0555
3.4670	1.2433	0.7390	-0.3025
3.8360	1.3444	0.4020	-0.9113
3.0160	1.1039	0.8940	-0.1120
3.7060	1.3100	0.7770	-0.2523
3.5830	1.2762	0.6520	-0.4277
3.3300	1.2030	0.7290	-0.3160
4.0320	1.3943	0.7360	-0.3065
3.1260	1.1398	0.7810	-0.2472

Table 6: Differences in the Period Averages Withand Without Missing Values

Period i	$\bar{X}_{i.(1)}$	$\bar{X}_{i.(2)}$	$\bar{X}_{i.(1)}$ - $\bar{X}_{i.(2)}$
1	3.6510	3.6510	0
2	3.3190	3.3190	0
3	3.1600	3.1600	0
4	3.4150	3.4150	0
5	3.4050	3.4000	0.0050
6	3.7550	3.7550	0
7	3.2990	3.2901	0.0089
8	4.0140	4.0111	0.0029
9	3.1700	3.1700	0

Table 7: Differences in the Seasonal AveragesWithand Without Missing Values

$Season\ j$	$\bar{X}_{.j(1)}$	$\bar{X}_{.j(2)}$	$\bar{X}_{.j(1)} - \bar{X}_{.j(2)}$
1	3.8670	3.8670	0
2	3.2470	3.2470	0
3	3.2700	3.2700	0
4	3.1030	3.1011	0.0019
5	3.4670	3.4670	0
6	3.8360	3.8360	0
7	3.0160	3.0160	0
8	3.7060	3.7060	0
9	3.5830	3.5822	0.0008
10	3.3300	3.3300	0
11	4.0320	4.0303	0.0017
12	3.1260	3.1260	0

Table 8: Differences in the Periodic Standard Deviations With and Without Missing Values

$Period\ i$	$\hat{\sigma}_{i.(1)}$	$\hat{\sigma}_{i.(2)}$	$\hat{\sigma}_{i.(1)} - \hat{\sigma}_{i.(2)}$
1	0.3335	0.3335	0
2	0.5810	0.5810	0
3	0.6360	0.6360	0
4	1.0490	1.0490	0
5	0.8630	0.8613	0.0017
6	0.8770	0.8770	0
7	0.8920	0.8911	0.0009
8	0.4340	0.4309	0.0031
9	1.0520	1.0520	0

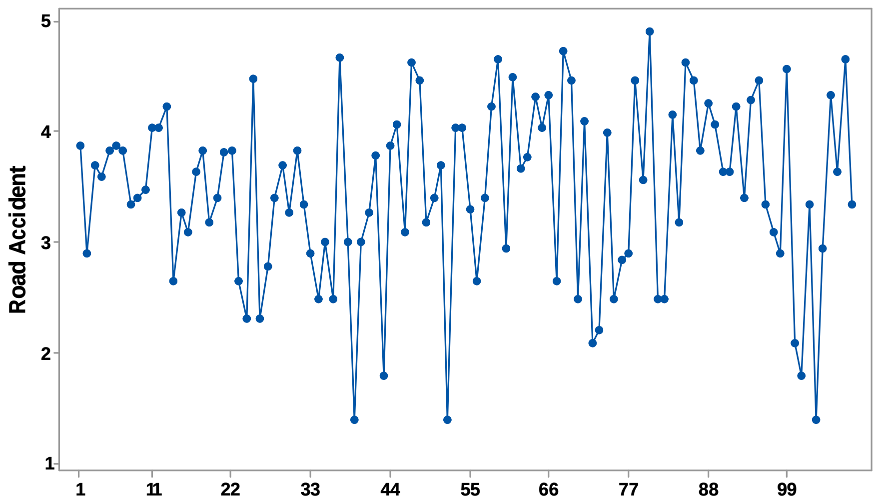

Fig 3.2: Time plot of transformed series with missing observations

Table 9: Differences in the Seasonal Standard Deviations Withand Without Missing Values

Season j	$\hat{\sigma}_{.j(1)}$	$\hat{\sigma}_{.j(2)}$	$\hat{\sigma}_{.j(1)}$ - $\hat{\sigma}_{.j(2)}$
1	0.8620	0.8620	0
2	0.6880	0.6880	0
3	0.9350	0.9350	0
4	0.9460	0.9423	0.0037
5	0.7390	0.7390	0
6	0.4020	0.4020	0
7	0.8940	0.8940	0
8	0.7770	0.7770	0
9	0.6520	0.6512	0.0008
10	0.7290	0.7290	0
11	0.7360	0.7355	0.0005
12	0.7810	0.7810	0

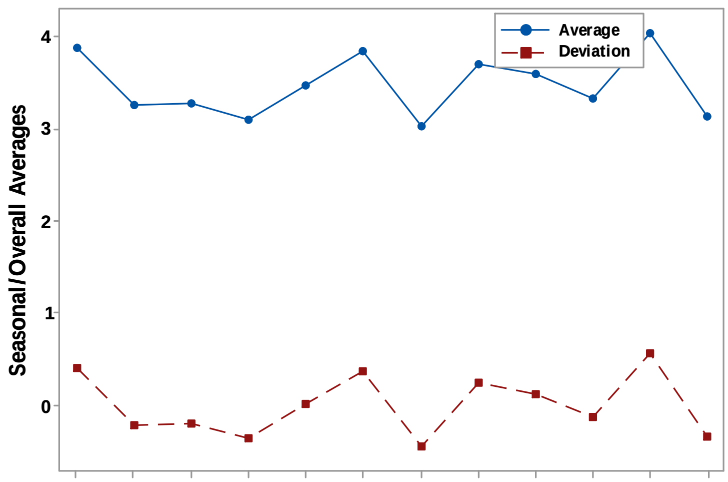

Fig 3.3: Time plot of seasonal and overall averages

Comment [Ma4]: Need some explanation regarding the Fig 3.3. Deviation stands for reflection of average or what?

3.3 Estimation of Transformed Missing Observations

Using (8)

$$Transformed\ \ value\ \ \hat{X}_{5,4} = 3.7062 + (-0.0320)[(5-1)12+4] + 0.0003[(5-1)12+4]^2 + (-0.0366)$$

$$= 2.8534$$

$$Transformed\ \ value\ \ \hat{X}_{7,9} = 3.7062 + (-0.0320)[(7-1)12+9] + 0.0003[(7-1)12+9]^2 + (-0.1922)$$

$$= 2.8903$$

$$Transformed\ \ value\ \ \hat{X}_{8,11} = 3.7062 + (-0.0320)[(8-1)12+11] + 0.0003[(8-1)12+11]^2 + (-0.3951)$$

$$= 2.9786$$

3.4 Estimation of Untransformed Missing Observations

Using (9)

$$Untransformed\ \ value\ \ \hat{X}_{5,4} = e^{2.8534} = 17.3466 \approx 17$$

$$Untransformed\ \ value\ \ \hat{X}_{7,9} = e^{2.8903} = 17.9987 \approx 18$$

$$Untransformed\ \ value\ \ \hat{X}_{8,11} = e^{2.9786} = 19.6602 \approx 20$$

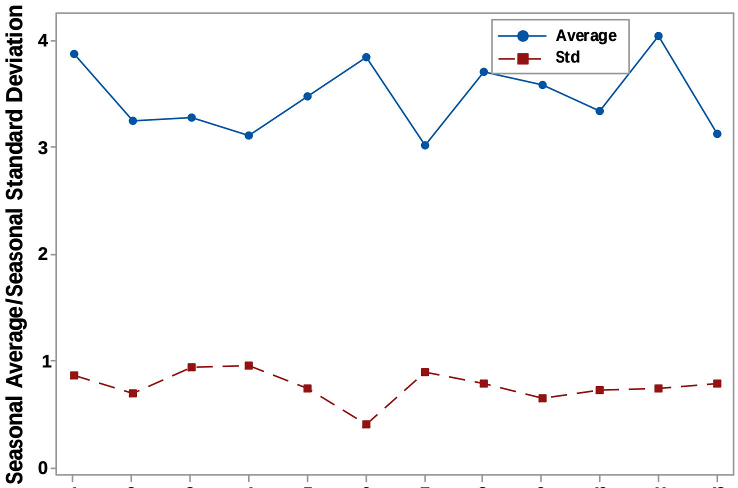

Fig 3.4: Time plot of seasonal averages and standard deviations

Table 10: The Deviations of Seasonal Averages from the Overall Averages $\bar{X}_{.j} - \bar{X}_{..}$

Seasons j	$\bar{X}_{.j}$	$\bar{X}_{.j} - \bar{X}_{..}$
1	3.8670	0.4017
2	3.2470	-0.2183
3	3.2700	-0.1953
4	3.1030	-0.3623
5	3.4670	0.0017
6	3.8360	0.3707
7	3.0160	-0.4493
8	3.7060	0.2407
9	3.5830	0.1177
10	3.3300	-0.1353
11	4.0320	0.5667
12	3.1260	-0.3393

From the transformed series listed in Appendix B, the periodic and seasonal averages in the presence and absence of missing values are listed in Tables 4 and 5. The difference in the periodic and seasonal averages are given in Tables 6 and 7. Also, the differences the periodic and seasonal standard deviations are shown in Tables 8 and 9. The seasonal averages are plotted again the seasonal standard deviation given in figure 3.3 and deviations of seasonal averages from the overall averages shown in figure 3.4. The time plots indicate no significant increase or decrease relatives to any increase or decrease in the seasonal averages, which suggest additive model.The comparison of the standard averages in Table 7 and seasonal standard deviations in Table 9 in the presence and absence of missing values show that, the are approximately same. Therefore, there are no effect inthe Buys-Ballot estimates of time series components.

4. Concluding Remarks

This study has examined the effect of seasonal averages and seasonal standard deviations on Buys-Ballot estimates of time series components in the presence of missing values. The emphasis is to determine the relationship between seasonal averages and seasonal standard deviations with and without missing values. Results of this analysis show that,1) the differences between seasonal averages and seasonal standard deviations with and without missing values have insignificant effect, because they are approximated the same. 2) the seasonal averages and standard deviation and the deviations of seasonal averages from the overall averages listed in figures 3.3 and 3.4 indicate no significant increase or decrease relative to any increase or decrease in the seasonal averages, which suggest that, the model for decomposition is additive.

Comment [Ma5]: I don't find any result for this one. Author need polish the writing so that it is understandable and fulfil the objective of this study.

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Appendix A: Table for the actual data on number of road traffic accidents with missing values(2013-2020)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2013	48	13	40	36	46	48	46	28	30	32	56	56	40.33	138.06
2014	68	14	26	22	38	46	24	30	45	46	14	10	31.92	293.54
2015	88	10	16	30	40	26	46	28	18	12	20	12	28.83	473.79
2016	106	20	4	20	26	44	6	48	58	22	102	86	45.20	1288.30
2017	24	30	40	-	56	56	27	14	30	68	105	19	39.42	777.72
2018	89	39	43	74	56	76	14	112	87	12	60	8	55.83	1126.15
2019	9	54	12	17	18	86	35	134	-	12	63	24	39.70	1480.2
2020	101	86	46	70	58	38	38	68	30	72	-	28	60.08	584.81
2021	22	18	96	8	6	28	4	19	76	38	105	28	37.30	1235.30

$\overline{X}_{.j}$	61.70	32.11	35.89	31.22	38.22	49.78	26.67	53.40	42.89	34.89	67.90	30.11		
$\sigma^2_{.j}$	1368.3	599.61	735.11	631.44	335.44	408.44	256.75	1843.3	668.36	544.11	1208.9	648.11		

Appendix B: Table for the transformed series on number of road traffic accidents with missing values (2013-2021)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\overline{X}_{i.}$	$\sigma^2_{i.}$
2013	3.87	3.90	2.89	3.67	3.58	3.82	3.87	3.82	3.33	3.40	4.03	4.03	3.65	0.11
2014	4.22	2.64	3.26	3.09	3.64	3.83	3.17	3.40	3.80	3.81	3.83	2.64	3.32	0.34
2015	4.48	2.30	2.77	3.40	3.69	3.26	3.83	3.33	2.89	2.48	2.99	2.48	3.16	0.41
2016	4.66	2.99	1.39	2.99	3.26	3.78	1.79	3.87	4.06	3.09	4.62	4.45	3.42	1.10
2017	3.18	3.40	3.69	-	4.03	4.03	3.29	2.64	3.40	4.22	4.65	2.94	3.41	0.73
2018	4.49	3.66	3.76	4.30	4.03	4.33	2.64	4.72	2.48	2.48	4.09	2.08	3.76	0.77
2019	2.20	3.99	2.48	2.83	2.89	4.45	3.56	4.90	-	2.48	4.14	3.18	3.30	0.80
2020	4.62	4.45	3.82	4.25	4.06	3.64	3.64	4.22	3.40	4.28	-	3.33	4.01	0.19
2021	3.09	2.89	4.56	2.08	1.79	3.33	1.39	2.94	4.33	3.64	4.65	3.33	3.17	1.11
$\overline{X}_{.j}$	3.87	3.25	3.27	3.10	3.47	3.84	3.02	3.71	3.58	3.33	4.03	3.13		
$\sigma^2_{.j}$	0.74	0.47	0.87	0.89	0.55	0.16	0.80	0.60	0.43	0.53	0.54	0.61		

Appendix C: Table for the actual data on number of road traffic accidents without missing values(2013-2021)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\overline{X}_{i.}$	$\sigma^2_{i.}$
2013	48	13	40	36	46	48	46	28	30	32	56	56	40.33	138.06
2014	68	14	26	22	38	46	24	30	45	46	14	10	31.92	293.54
2015	88	10	16	30	40	26	46	28	18	12	20	12	28.83	473.79
2016	106	20	4	20	26	44	6	48	58	22	102	86	45.20	1288.30
2017	24	30	40	35	56	56	27	14	30	68	105	19	39.42	777.72
2018	89	39	43	74	56	76	14	112	87	12	60	8	55.83	1126.15

2019	9	54	12	17	18	86	35	134	85	12	63	24	39.70	1480.2
2020	101	86	46	70	58	38	38	68	30	72	51	28	60.08	584.81
2021	22	18	96	8	6	28	4	19	76	38	105	28	37.30	1235.30
$\bar{X}_{.j}$	61.70	32.11	35.89	31.22	38.22	49.78	26.67	53.40	42.89	34.89	67.90	30.11		
$\sigma^2_{.j}$	1368.3	599.61	735.11	631.44	335.44	408.44	256.75	1843.3	668.36	544.11	1208.9	648.11		

Appendix D: Table for the transformed series on number of road traffic accidents without missing values (2013-2021)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2013	3.87	3.90	2.89	3.67	3.58	3.82	3.87	3.82	3.33	3.40	4.03	4.03	3.65	0.11
2014	4.22	2.64	3.26	3.09	3.64	3.83	3.17	3.40	3.80	3.81	3.83	2.64	3.32	0.34
2015	4.48	2.30	2.77	3.40	3.69	3.26	3.83	3.33	2.89	2.48	2.99	2.48	3.16	0.41
2016	4.66	2.99	1.39	2.99	3.26	3.78	1.79	3.87	4.06	3.09	4.62	4.45	3.42	1.10
2017	3.18	3.40	3.69	1.38	4.03	4.03	3.29	2.64	3.40	4.22	4.65	2.94	3.41	0.73
2018	4.49	3.66	3.76	4.30	4.03	4.33	2.64	4.72	2.48	2.48	4.09	2.08	3.76	0.77
2019	2.20	3.99	2.48	2.83	2.89	4.45	3.56	4.90	2.48	2.48	4.14	3.18	3.30	0.80
2020	4.62	4.45	3.82	4.25	4.06	3.64	3.64	4.22	3.40	4.28	4.45	3.33	4.01	0.19
2021	3.09	2.89	4.56	2.08	1.79	3.33	1.39	2.94	4.33	3.64	4.65	3.33	3.17	1.11
$\bar{X}_{.j}$	3.87	3.25	3.27	3.10	3.47	3.84	3.02	3.71	3.58	3.33	4.03	3.13		
$\sigma^2_{.j}$	0.74	0.47	0.87	0.89	0.55	0.16	0.80	0.60	0.43	0.53	0.54	0.61		