Original Research Article

CAPACITY CONSTRAINED WAREHOUSE INVENTORY MODEL FOR DETERIORATING ITEMS UNDER TWO - LEVEL PARTIAL TRADE CREDIT.

ABSTRACT

In trade credit financing, failure in payment leads to great loss or total collapse of business. Thus, good policies are invented to address situations when retailers or customers are suspected to be credit – risk in order to reduce the effect of the failure in payments on the business. In most cases, the focus has been on either credit - risk retailer or credit – risk customers but not both. This has not taken care of some market happenings when both retailers and customers are simultaneously perceived as credit – risk. This provides additional dimension to the supplier and retailer to curtail the menace of failure in payment at upstream and downstream levels respectively. In this study, partial trade credit is proposed concurrently at both upstream and downstream levels to curtail the menace of failure in payment. Numerical illustration of the model developed for the situation is given. The model determines the optimal cost in each of the possible cases and sensitivity analysis carried out to see the effect of parameter changes on the optimal solution.

Keywords: Partial Trade credit, Deterioration, Credit worthy, Credit – risk.

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1. INTRODUCTION

Competition in the market necessitates invention of strategies or procedures such as promotional tools to enhance sales so as to increase earning. One of such promotional tools, among others is trade credit financing. The trade credit financing, as acknowledged in the literature, such as Goyal (1985), Teng (2002) and Teng (2009), has the advantage of stimulating the demand of the retailers and/or the customers. It serves in some instances, as alternative to price or cash discount. It also helps in reducing the on – hand inventory level thereby reducing the holding cost.

Goyal (1985) was the first to come up with the idea of trade credit but considered it to be between the wholesaler/supplier and the retailers (upstream) but did not extended it to the customers. This, in some instances, has not always been the case. The retailer too may decide to adopt same means of stimulating the demand of his/her customers, hence the need to extend the grace of the permissible delay in payment to the customers (downstream), which brings in to the literature, the work of Huang (2003).

In trade credit, one of the most fundamental aspects to give much attention to, is failure in payment. Failure in payment is a situation when the benefiter of the trade credit facility does not turn up for the payment of goods ordered at the agreed period. This has a big negative effect when planning a business transaction. The effect of the failure in payment accumulates with time and usually leads to partial and in some cases total collapse of a business. As a result of this, many policies were invented to curtail the menace. One of the policies adopted is partial trade credit. Partial trade credit is a policy that attaches condition of depositing some amount of money immediately after receipt of the delivery which will cover some proportion of the goods before benefitting from the trade credit on the remaining proportion of the goods. In other words, instead of offering the retailer/customer trade credit on the whole order, the supplier/retailer comes up with pre – specified quantity of goods that must be paid before benefitting from the facility, which to the belief of the party in play, in case of breach, the default in payment will have little or no impact on the business.

In most of the contributions in the literature, it is always assumed that the customers are not credit worthy, so the retailer gives them the partial trade credit known as downstream. Some of such work in the literature include Teng (2009) who considers retailer offering distinct trade credit policy to the good and bad customers. Wu et al (2016) developed the model for the retailer who assumed the customers as credit – risk and therefore offers them with partial trade credit. Yang (2019) also developed an inventory models but linking the trade credit (downstream) to order size in order to curtail the menace of default in payment. There are also some few contributions in the literature that assumed the retailer to be credit – risk whereas the retailer's customers are not. Nita et al (2015) for instance looked at the inventory situation where the supplier offers the retailer with order – linked trade credit or cash discount. Trade credit period (fixed) for regular order size or cash discount.

So far, in the literature, a situation where both retailer and customers are assumed not credit worthy has not been reported. However, situation could arise where the retailer is considered credit risk and the retailer also considers the customers as credit risk. This is the situation addressed in this paper.

As a result of permissible delay in payment, the retailer may decide to order and stock huge quantity of goods so as to earn much profit over the sales he/she will make during the allowed period. This could also happen if the retailer fears about the scarcity of the item in the near future or if the item is seasonal. The retailer might have excess after exceeding the maximum stocking capacity of his/her own warehouse(OW) which will necessitate renting another warehouse (RW) with unlimited capacity. For the authors who worked on two – warehouses with condition of permissible delay in payment; see Bhunia and Maiti (1998), Lee (2006), Lin and Lin (2007), Liang and Zhou (2011), Yang and Chang (2013), Bhunia et al (2014) and many others.

In this study, we consider a situation where the retailer is having capacity constrained own warehouse and also both the retailer and the retailer's customers are assumed credit – risk. We propose a two – level partial trade credit to take care of the perceived failure in payment. That is, supplier offers partial permissible delay in payment to the retailer and on the same business, the retailer passes the grace to the customer but requesting them to deposit some amount immediately after the delivery of the consignment.

The structure of this work is, notations and assumptions in section 2, model formulation in section 3, derivation of the costs quantities in section 4, optimization and analysis in section 5, numerical examples and sensitivity analysis in section 6 whereas summary and conclusion are in section 7.

2. **NOTATIONS AND ASSUMPTIONS**

The following are the notations used in the model

$I_r(t), I_o(t)$ –Inventorylevels of the *RW* and *OW* respectively at time *t*.

W – Maximumquantity of the item that can be stored in *OW*.

t_w– Timeat which inventory at *RW* drops to zero.

T– Replenishment cycle for the model.

α, β– Constantdeterioration rates at OW and RW respectively.

h_r, h_o– holdingcosts per unit per unit time excluding interest charges of *RW* and *OW* respectively with$h_r \geq h_o$.

A– Orderingcost per order.

I_P ,I_e– Interestpayable and interest earned respectively.

c, *p*– purchasingcost and selling price per unit of the item respectively.

M, N– tradecredit period offered to the retailer and customers respectively.

δ, γ– Respectiveproportions of goods ordered to be paid by the retailer and customers before benefitting from the trade credit on the remaining $(1-\delta)$ and $(1-\gamma)$ proportions.

RW – Rented warehouse

OW – Own warehouse

TC – totalrelevant costs of the model to be minimized.

All other notations not defined here will be defined in due course.

The following are assumptions made in building the model

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a. The model considers partial trade credit financing, that is, partial upstream for the retailer and partial downstream for the customers.
b. The demand rate is the same in both warehouses.
c. Deterioration rates are constant in both warehouses but different.
d. Deterioration in *RW* is less than that in *OW*, i.e. $\alpha > \beta$ due to higher preserving facilities in *RW* and charges higher holding cost in *RW* than in *OW*, i.e. $h_r > h_o$. This gives rise to the assumption $h_r - h_o > c(\alpha - \beta)$.
e. The demand rate in the warehouse is greater than the deterioration rate at the warehouse. Thus, $D > \alpha W$ and $D > \beta(Q - W)$ for OW and RW respectively. Where *Q* is the total quantity of the goods ordered.
f. The proportion to be paid before giving the trade credit shall not exceed the proportion of the items given on trade credit, i.e. $\delta \ngtr (1 - \delta)$.
g. Interest payable is assumed to be greater than interest earned by the retailer.
h. The dispatching policy is last – in first – out *LIFO* due to economic reasons.
i. Shortage is not allowed and lead time is assumed to be zero.
j. Also, we restrict $M > N$ and $N \leq t_w$.

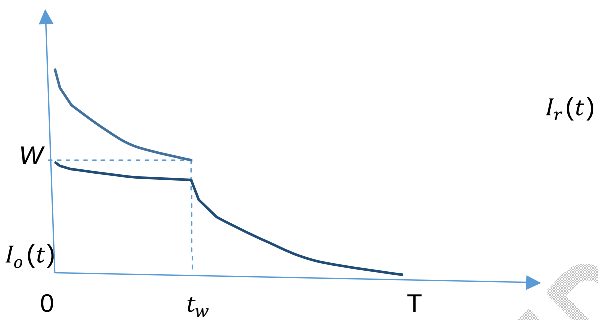

Fig 1: Graphical representation of the model.

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3. **MATHEMATICAL MODEL FORMULATION**

Since the dispatching policy is first in last out, then, during the period $t \in (0, t_w)$, sales and deterioration take place in RW and therefore, depletion of inventory is due to combined effect of demand and deterioration. At $t = t_w$, the inventory at RW drops to zero. For OW during this same period, only deterioration effect depletes the inventory. These phenomena are represented by the following differential equations:

$$\frac{dI_r(t)}{dt} + \beta I_r(t) = -D \,, \qquad 0 \leq t \leq t_w \tag{1}$$

$$\frac{dI_o(t)}{dt} + \alpha I_o(t) = 0 \quad , \qquad 0 \leq t \leq t_w \tag{2}$$

With boundary condition $I_r(t_w) = 0$ and initial condition $I_o(0) = W$ for equations (1) and (2) respectively.

During the period $t \in (t_w, T)$, no activity in RW as goods are finished, sales and deterioration take place in OW and therefore, depletion of inventory is due to combined effect of demand and deterioration. At $t = T$, the inventory at OW drops to zero. Meaning that both warehouses are empty. This is represented by the following differential equation:

$$\frac{dI_o(t)}{dt} + \alpha I_o(t) = -D \quad , \qquad t_w \leq t \leq T \tag{3}$$

With boundary condition $I_o(T) = 0$

The solutions of the equations (1) - (3) are respectively given as

$$I_r(t) = \frac{D}{\beta}\left(e^{\beta(t_w - t)} - 1\right) 0 \leq t \leq t_w \tag{4}$$

$$I_o(t) = We^{-\alpha t} 0 \leq t \leq t_w \tag{5}$$

$$I_o(t) = \frac{D}{\alpha}\left(e^{\alpha(T-t)} - 1\right) t_w \le t \le T \tag{6}$$

To establish continuity in OW, we substitute $t = t_w$ in equations (5) and (6) to obtain

$$T = t_w + \frac{1}{\alpha}\ln\left(1 + \frac{\alpha W e^{-\alpha t_w}}{D}\right) \tag{7}$$

3.1 Derivation of the cost quantities

The holding cost(HC) per cycle for the model is given by

$$\text{HC}= \frac{Dh_r}{\beta^2}\left(e^{\beta t_w} - \beta t_w - 1\right) + \frac{Wh_o}{\alpha}\left(1 - e^{-\alpha t_w}\right) + \frac{Dh_o}{\alpha^2}\left(e^{\alpha(T-t_w)} - \alpha(T - t_w) - 1\right) \tag{8}$$

Likewise, the deterioration cost (DC)per cycle for the model is given as

$$\text{DC}= \frac{cD}{\beta}\left(e^{\beta t_w} - \beta t_w - 1\right) + cW\left(1 - e^{-\alpha t_w}\right) + \frac{cD}{\alpha}\left(e^{\alpha(T-t_w)} - \alpha(T - t_w) - 1\right) \tag{9}$$

The ordering cost (OC)per order OC is A (10)

To get the total inventory cost per cycle, we add up the relevant costs, that is, holding, deterioration and ordering costs plus the interest incurred by the retailer minus the interest earned.

For the interest payable and the interest earned by the retailer, the following scenarios are the possibilities based on the values of N, M, t_w and T.

(i) $N < M \le t_w < T$, (ii) $N \le t_w \le M < T$, (iii) $T \le M < T + N$ and (iv) $T + N \le M$.

Case 1: $N < M \le t_w < T$

In this case, the retailer paid for δ proportion of the goods and has been given trade credit on the remaining $(1-\delta)$ proportion of the total goods ordered for *M* days. The retailer gives customers trade credit period *N* on $1-\gamma$ portion after paying for γ portion of the goods sold to them. Therefore, at expiration of the given trade credit period *M*, the retailer pays interest on goods sold to customers on cash and credit basis from $1-\delta$ proportion and the interest payable is given by

$$I_{P1} = (1-\delta)cI_p\left(\gamma\left(\int_M^{t_w} I_r(t)dt + \int_M^{T} I_o(t)dt\right) + (1-\gamma)\left(\int_M^{t_w+N} I_r(t)dt + \int_M^{T+N} I_o(t)dt\right)\right)$$

Using equations (4), (5) and (6) and after simplifications, we get the interest payable as

$$I_{P1} = (1-\delta)cI_p\Bigg(\frac{D}{\beta^2}\left(\gamma\left(e^{\beta(t_w-M)} - \beta(t_w - M) - 1\right) + (1-\gamma)\left(e^{\beta(t_w-M)} - \beta(t_w + N - M) - e^{-\beta N}\right)\right) + \frac{W}{\alpha}\left(e^{-\alpha M} - e^{-\alpha t_w}\right) + \frac{D}{\alpha^2}\left(\gamma\left(e^{\alpha(T-t_w)} - \alpha(T - t_w) - 1\right) + (1-\gamma)\left(e^{\alpha(T-t_w)} - \alpha(T + N - t_w) - e^{-\alpha N}\right)\right)\Bigg) \tag{11}$$

Likewise, the retailer will make sales and earn interest on δ proportion of goods ordered during the period $(0, T)$ from γ portion and during the period (N, T) from $(1-\gamma)$ portion sold on cash and credit basis respectively. Also, from $1-\delta$ proportion of the goods, the retailer will earn interest on sales made during the periods $(0, M)$ from γ portion and during the period(N, M) from $1-\gamma$ portion. Therefore, the interest earned by the retailer in this case is given as

$$I_{E1} = pI_e\left(\delta\left(\gamma\int_0^T Dtdt + (1-\gamma)\int_N^T D(t-N)dt\right) + (1-\delta)\left(\gamma\int_0^M Dtdt + (1-\gamma)\int_N^M D(t-N)dt\right)\right)$$

After simplification, we obtain

$$I_{E1} = \frac{1}{2}pDI_e\left(\delta\left(\gamma T^2 + (1-\gamma)(T-N)^2\right) + (1-\delta)\left(\gamma M^2 + (1-\gamma)(M-N)^2\right)\right) \tag{12}$$

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For this case, the total annual inventory cost is given by

$$TC1 = \frac{1}{T}(OC + HC + DC + I_{P1} - I_{E1})$$

Using equations (8), (9), (10), (11) and (12), we see that

$$TC1 = \frac{1}{T}\Bigg(A + \frac{D}{\beta^2}\Big((h_r + c\beta)\big(e^{\beta t_w} - \beta t_w - 1\big) + (1-\delta)cI_p\Big(\gamma\big(e^{\beta(t_w-M)} - \beta(t_w - M) - 1\big) + (1 - \gamma)\big(e^{\beta(t_w-M)} - \beta(t_w + N - M) - e^{-\beta N}\big)\Big)\Big) + \frac{W}{\alpha}\Big((h_o + c\alpha)(1 - e^{-\alpha t_w}) + (1-\delta)cI_p(e^{-\alpha M} - e^{-\alpha t_w})\Big) + \frac{D}{\alpha^2}\Big((h_o + c\alpha + (1-\delta)\gamma cI_p)\big(e^{\alpha(T-t_w)} - \alpha(T - t_w) - 1\big) + (1-\delta)(1-\gamma)cI_p\big(e^{\alpha(T-t_w)} - \alpha(T + N - t_w) - e^{-\alpha N}\big)\Big) - \frac{1}{2}pDI_e\big((1-\delta)(\gamma M^2 + (1-\gamma)(M-N)^2) + \delta(\gamma T^2 + (1-\gamma)(T-N)^2)\big)\Bigg) \tag{13}$$

Case 2: $N \le t_w \le M < T$

In this case, the trade credit period given to the retailer exceeds the period of which goods in RW finish. Thus, the retailer will pay interest for only unsold goods in OW from $(1-\delta)$ proportion ordered up to the time the last customer will settle his/her account. But the retailer has to pay interest on the outstanding payment from the goods sold on credit basis from RW for the period$(M,\ t_w + N)$ if and only if $M \le t_w + N$. Therefore, the interest payable by the retailer in this case is given by

$$I_{P2} = (1-\delta)cI_p\left(\gamma\int_M^T I_o(t)dt + (1-\gamma)\left(\int_M^{t_w+N} I_r(t)dt + \int_M^{T+N} I_o(t)dt\right)\right)$$

Using equations (4) and (6), we get

$$I_{P2} = (1-\delta)cI_p\Big(\gamma\frac{D}{\alpha^2}\big(e^{\alpha(T-M)} - \alpha(T-M) - 1\big) + (1-\gamma)\Big(\frac{D}{\beta^2}\big(e^{\beta(t_w-M)} - \beta(t_w + N - M) - e^{-\beta N}\big) + \frac{D}{\alpha^2}\big(e^{\alpha(T-M)} - \alpha(T + N - M) - e^{-\alpha N}\big)\Big)\Big) \tag{14}$$

On the other hand, the retailer will earn interest in two ways; firstly, on δ proportion of the goods during the period $(0,T)$ from γ portion and during the period (N,T) from $1-\gamma$ portion sold on cash and credit basis respectively. Secondly, on $1-\delta$ proportion, during the periods $(0,M)$ from γ portion and during the period (N,M) from $1-\gamma$ portion of the goods sold on cash and credit basis respectively. Thus, the interest earned by the retailer is given as

$$I_{E2} = pI_e\left(\delta\left(\gamma\int_0^T Dtdt + (1-\gamma)\int_N^T D(t-N)dt\right) + (1-\delta)\left(\gamma\int_0^M Dtdt + (1-\gamma)\int_N^M D(t-N)dt\right)\right)$$

After simplification, we obtain

$$I_{E2} = \frac{1}{2}pDI_e\big(\delta(\gamma T^2 + (1-\gamma)(T-N)^2) + (1-\delta)(\gamma M^2 + (1-\gamma)(M-N)^2)\big) \tag{15}$$

The total annual relevant cost for the model in this case is given by

$$TC2 = \frac{1}{T}(OC + HC + DC + I_{P2} - I_{E2})$$

Using equations (8), (9), (10), (14) and (15), we see that

$$TC2 = \frac{1}{T}\Bigg(A + \frac{D}{\beta^2}\Big((h_r + c\beta)\big(e^{\beta t_w} - \beta t_w - 1\big) + (1-\delta)(1-\gamma)cI_p\big(e^{\beta(t_w-M)} - \beta(t_w + N - M) - e^{-\beta N}\big)\Big) + \frac{W}{\alpha}(h_o + c\alpha)(1 - e^{-\alpha t_w}) + \frac{D}{\alpha^2}\Big((h_o + c\alpha)\big(e^{\alpha(T-t_w)} - \alpha(T - t_w) - 1\big) + (1-\delta)cI_p\big(\gamma\big(e^{\alpha(T-M)} -$$

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$$\alpha(T-M)-1)+(1-\gamma)\left(e^{\alpha(T-M)}-\alpha(T+N-M)-e^{-\alpha N}\right)\Big)\Big)-\frac{1}{2}pDI_e\Big(\delta\big(\gamma T^2+(1-\gamma)(T-N)^2\big)+(1-\delta)\big(\gamma M^2+(1-\gamma)(M-N)^2\big)\Big)\Big) \tag{16}$$

Case 3: $T \leq M < T + N$

In this scenario, goods are finished in both warehouses, therefore, the retailer will only pay interest on the outstanding balance of the goods sold to the customers on credit basis from $1-\delta$ proportion of the goods ordered. Therefore, the interest payable is given by

$$I_{p3} = (1-\delta)cI_p\left((1-\gamma)\int_M^{T+N} I_o(t)dt\right)$$

Using equation (6), we obtain

$$I_{p3} = (1-\delta)(1-\gamma)cI_p\left(\frac{D}{\alpha^2}\left(e^{\alpha(T-M)}-\alpha(T+N-M)-e^{-\alpha N}\right)\right) \tag{17}$$

Likewise, the retailer will earn interest from sales made on δ proportion of the goods during the period $(0, M)$ from γ portion sold to customers on cash basis and during the period (N, M) from $1-\gamma$ portion of the goods sold to customers on credit basis. Also, the retailer will earn interest on $1-\delta$ proportion during the periods $(0, M)$ and (N, M) from the portions γ and $1-\gamma$ for the goods sold to customers respectively. Therefore, interest earned by the retailer is given as

$$I_{E3} = pI_e\left(\delta\left(\gamma\int_0^T Dtdt+(1-\gamma)\int_N^T D(t-N)dt+DT(M-T)\right)\right.$$
$$\left.+(1-\delta)\left(\gamma\int_0^T Dtdt+(1-\gamma)\int_N^T D(t-N)dt+DT(M-T)\right)\right)$$

Note that, $DT(M-T)$ is the value of revenue generated on *DT* during the period$[M\ ,T]$.

Simplifying, we get

$$I_{E3} = \frac{1}{2}pDI_e\left(\gamma T^2+(1-\gamma)(T-N)^2+2T(M-T)\right) \tag{18}$$

Therefore, the total annual relevant cost is given by

$$TC3 = \frac{1}{T}(OC+HC+DC+I_{P3}-I_{E3})$$

Using equations (8), (9), (10), (17) and (18), we see that

$$TC3 = \frac{1}{T}\Big(A+\frac{D}{\beta^2}(h_r+c\beta)\left(e^{\beta t_w}-\beta t_w-1\right)+\frac{W}{\alpha}(h_o+c\alpha)\left(1-e^{-\alpha t_w}\right)+\frac{D}{\alpha^2}\Big((h_o+c\alpha)\big(e^{\alpha(T-t_w)}-\alpha(T-t_w)-1\big)+(1-\delta)(1-\gamma)cI_p\left(e^{\alpha(T-M)}-\alpha(T+N-M)-e^{-\alpha N}\right)\Big)-\frac{1}{2}pDI_e\left(\gamma T^2+(1-\gamma)(T-N)^2+2T(M-T)\right)\Big) \tag{19}$$

Case 4: $T + N \leq M$

In this case, goods are finished and the retailer has no outstanding payment from all the goods sold on cash and credit basis. Consequently, interest payable by the retailer is given as

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$$I_{P4} = 0 \tag{20}$$

However, the retailer will earn interest from sales made on δ proportion of the goods during the period $(0, M)$ from γ portion sold to customers on cash basis and the period (N, M) from $1-\gamma$ portion of the goods sold to customers on credit basis. Also, the retailer will earn interest on $1-\delta$ proportion during the period $(0, M)$ and during the period (N, M) from portions γ and $1-\gamma$ sold to customers respectively. Therefore, the interest earn by the retailer is given as

$$I_{E4} = pI_e\left(\delta\left(\gamma\left(\int_0^T Dtdt + DT(M-T)\right)\right.\right.$$
$$\left.+ (1-\gamma)\left(\int_N^T D(t-N)dt + DT(T+N-T) + D(T+N)(M-T-N)\right)\right)$$
$$+ (1-\delta)\left(\gamma\left(\int_0^T Dtdt + DT(M-T)\right)\right.$$
$$\left.\left.+ (1-\gamma)\left(\int_N^T D(t-N)dt + DT(T+N-T) + D(T+N)(M-T-N)\right)\right)\right)$$

Note that, $DT(T+N-T)$ is the value of revenue generated on the amount *DT* during the period $[T,\ T+N]$ whereas $D(T+N)(M-T-N)$ is revenue generated on *D(T+N)* during the period$[T+N, M]$.

Simplifying, we have

$$I_{E4} = \frac{1}{2}pDI_e\left(\gamma\left(T^2 + 2T(M-T)\right) + (1-\gamma)\left((T-N)^2 + 2TN + 2(T+N)(M-T-N)\right)\right) \tag{21}$$

Therefore, the total annual relevant cost is given by

$$TC4 = \frac{1}{T}(OC + HC + DC + I_{P4} - I_{E4})$$

Using equations (8), (9), (10), (20) and (21), we get

$$TC4 = \frac{1}{T}\left(A + \frac{D}{\beta^2}(h_r + c\beta)\left(e^{\beta t_w} - \beta t_w - 1\right) + \frac{W}{\alpha}(h_o + c\alpha)\left(1 - e^{-\alpha t_w}\right) + \frac{D}{\alpha^2}\left((h_o + c\alpha)\left(e^{\alpha(T-t_w)} - \alpha(T-t_w) - 1\right)\right) - \frac{1}{2}pDI_e\left(\gamma\left(T^2 + 2T(M-T)\right) + (1-\gamma)\left((T-N)^2 + 2TN + 2(T+N)(M-T-N)\right)\right)\right) \tag{22}$$

4. **OPTIMIZATION AND ANALYSIS**

The necessary conditions for TC1 to have minimum are $\frac{\partial TC1}{\partial t_w} = 0$ and $\frac{\partial TC1}{\partial T} = 0$

Using (13), differentiating, simplifying and setting the resultant equation to zero, we get

$$\frac{\partial TC1}{\partial t_w} = \frac{1}{T}\left(\frac{D}{\beta}\left((h_r + c\beta)\left(e^{\beta t_w} - 1\right) + (1-\delta)cI_p\left(e^{\beta(t_w - M)} - 1\right)\right) + W\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{-\alpha t_w} + \frac{D}{\alpha}\left(h_o + c\alpha + (1-\delta)cI_p\right)\left(1 - e^{\alpha(T-t_w)}\right)\right) = 0 \tag{23}$$

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Also, using equation (13), differentiating and setting the result to zero, we see that

$$\frac{\partial TC1}{\partial T} = \frac{1}{T}\left(\frac{D}{\alpha}\left(h_o + c\alpha + (1-\delta)cI_p\right)\left(e^{\alpha(T-t_w)} - 1\right) - \delta p D I_e\left(T - (1-\gamma)N\right) - TC1\right) = 0 \qquad (24)$$

The solutions to Equations (23) and (24) give the values t_w and T for the model in this case. To confirm that the solutions exist and are unique, we show that the determinant of the Hessian matrix evaluated at (t_w^{1*}, T_1^*) is positive definite.

Using left hand side of equation (23), we get

$$\frac{\partial^2 TC1}{\partial t_w^2} = \frac{1}{T}\left(D\left((h_r + c\beta)e^{\beta t_w} + (1-\delta)cI_p e^{\beta(t_w - M)}\right) - \alpha W\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{-\alpha t_w} + D\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{\alpha(T-t_w)}\right) \qquad (25)$$

Evaluating equation (25) at (t_w^{1*}, T_1^*) we see that

$$\left.\frac{\partial^2 TC1}{\partial t_w^2}\right|_{(t_w^{1*},T_1^*)} = \frac{1}{T}\left(D\left((h_r + c\beta)e^{\beta t_w} + (1-\delta)cI_p e^{\beta(t_w-M)}\right) - \alpha W\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{-\alpha t_w} + D\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{\alpha(T-t_w)}\right)\Big|_{(t_w^{1*},T_1^*)} > 0$$ If and only if

$$D\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{\alpha(T-t_w)} - \alpha W\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{-\alpha t_w} > 0$$

Lemma 1: If $D > \alpha W$, then $D\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{\alpha(T-t)} > \alpha W\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{-\alpha t}$ for all values of *t*.

<u>Proof</u>

From the hypothesis $D - \alpha W > 0$. Also, $e^{\alpha T} > 1$

$\Longrightarrow (De^{\alpha T} - \alpha W)\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{-\alpha t} > 0$and the result follows.

Using left hand side of equation (24),

$$\frac{\partial^2 TC1}{\partial T^2} = \frac{1}{T}\left(D\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{\alpha(T-t_w)} - \delta p D I_e - 2\frac{\partial TC1}{\partial T}\right) \qquad (26)$$

Evaluating (26) at (t_w^{1*}, T_1^*) we have

$$\left.\frac{\partial^2 TC1}{\partial T^2}\right|_{(t_w^{1*},T_1^*)} = \frac{1}{T}\left(D\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{\alpha(T-t_w)} - \delta p D I_e - 2\frac{\partial TC1}{\partial T}\right)\Bigg|_{(t_w^{1*},T_1^*)}$$

and since from equation (24), $\left.\frac{\partial TC1}{\partial T}\right|_{(t_w^{1*},T_1^*)} = 0$ we see that

$$\left.\frac{\partial^2 TC1}{\partial T^2}\right|_{(t_w^{1*},T_1^*)} = \frac{1}{T}\left(D\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{\alpha(T-t_w)} - \delta p D I_e\right)\Big|_{(t_w^{1*},T_1^*)} > 0$$ if and only if

$$\frac{1}{T}\left(D\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{\alpha(T-t_w)} - \delta p D I_e\right) > 0$$

Lemma 2: if $(1-\delta)cI_p \geq \delta p I_e$ then the quantity given by $D\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{\alpha(T-t)} - \delta p D I_e > 0$ for all values of *t*<*T*.

<u>Proof</u>: Let $f(t) = D\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{\alpha(T-t)} - D\delta p I_e$.

If $t < T$, then$e^{\alpha(T-t)} > 1$. From the hypothesis, $(1-\delta)cI_p \geq \delta p I_e$ and so $f(t) > 0$.

Using left hand side of equation (23),

$$\frac{\partial^2 TC1}{\partial T \partial t_w} = -\frac{1}{T}\left(D\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{\alpha(T-t_w)} + \frac{\partial TC1}{\partial t_w}\right) \qquad (27)$$

Also, using left hand side of (24),

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$$\frac{\partial^2 TC1}{\partial t_w \partial T} = -\frac{1}{T}\left(D\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{\alpha(T-t_w)} + \frac{\partial TC1}{\partial t_w}\right) \tag{28}$$

Evaluating (27) and (28) at the point (t_w^{1*}, T_1^*) we see that

$$\left.\frac{\partial^2 TC1}{\partial T \partial t_w}\right|_{(t_w^{1*},T_1^*)} = -\frac{1}{T}\left(D\left(h_o + c\alpha + (1-\delta)cI_p\right)e^{\alpha(T-t_w)}\right)\Big|_{(t_w^{1*},T_1^*)} = \left.\frac{\partial^2 TC1}{\partial t_w \partial T}\right|_{(t_w^{1*},T_1^*)}$$

Theorem 1: if $(1-\delta)cI_p \geq \delta p I_e$, then the cost function TC1 in (13) is a convex function.

<u>Proof</u>

The proof follows from lemmas 1 and 2 and equations (25) – (28), since in that case $\left\{\frac{\partial^2 TC1}{\partial t_w^2}\frac{\partial^2 TC1}{\partial T^2} - \frac{\partial^2 TC1}{\partial t_w \partial T}\frac{\partial^2 TC1}{\partial T \partial t_w}\right\}\Big|_{(t_w^{1*},T_1^*)} > 0$ and $D > \alpha W$ from assumption (e).

The necessary conditions for TC2 to have minimum are $\frac{\partial TC2}{\partial t_w} = 0$ and $\frac{\partial TC2}{\partial T} = 0$

Using (16), differentiating and setting the result to zero, we find that

$$\frac{\partial TC2}{\partial t_w} = \frac{1}{T}\left(\frac{D}{\beta}\left((h_r + c\beta)\left(e^{\beta t_w} - 1\right) + (1-\delta)(1-\gamma)cI_p\left(e^{\beta(t_w - M)} - 1\right)\right) + W(h_o + c\alpha)e^{-\alpha t_w} + \frac{D}{\alpha}(h_o + c\alpha)\left(1 - e^{\alpha(T-t_w)}\right)\right) = 0 \tag{29}$$

Also, using equation (16), differentiating, simplification and setting the result to zero, we have

$$\frac{\partial TC2}{\partial T} = \frac{1}{T}\left(\frac{D}{\alpha}\left((h_o + c\alpha)\left(e^{\alpha(T-t_w)} - 1\right) + (1-\delta)cI_p\left(e^{\alpha(T-M)} - 1\right)\right) - \delta p D I_e(T - (1-\gamma)N) - TC2\right) = 0 \tag{30}$$

The solutions to Equations (29) and (30) give the values t_w and T for the model in this case. To confirm that the solutions exist and are unique, we show that the determinant of the Hessian matrix evaluated at (t_w^{2*}, T_2^*) is positive definite.

Using the left hand side of equation (29), we get

$$\frac{\partial^2 TC2}{\partial t_w^2} = \frac{1}{T}\left(D\left((h_r + c\beta)e^{\beta t_w} + (1-\delta)(1-\gamma)cI_p e^{\beta(t_w-M)}\right) - \alpha W(h_o + c\alpha)e^{-\alpha t_w} + D(h_o + c\alpha)e^{\alpha(T-t_w)}\right) \tag{31}$$

Evaluating (31) at (t_w^{2*}, T_2^*) we see that

$\left.\frac{\partial^2 TC2}{\partial t_w^2}\right|_{(t_w^{2*},T_2^*)} = \frac{1}{T}\left(D\left((h_r + c\beta)e^{\beta t_w} + (1-\delta)(1-\gamma)cI_p e^{\beta(t_w-M)}\right) - \alpha W(h_o + c\alpha)e^{-\alpha t_w} + D(h_o + c\alpha)e^{\alpha(T-t_w)}\right)\Big|_{(t_w^{2*},T_2^*)} > \frac{1}{T}D(h_o + c\alpha)e^{\alpha(T-t_w)} - \alpha W(h_o + c\alpha)e^{-\alpha t_w}\Big|_{(t_w^{2*},T_2^*)} = \frac{1}{T}\left((De^{\alpha T} - \alpha W)(h_o + c\alpha)e^{-\alpha t_w}\right)\Big|_{(t_w^{2*},T_2^*)} > 0$ since $e^{\alpha T} > 1$ and $D - \alpha W > 0$ from the assumption €.

Using left hand side of equation (30), we obtain

$$\frac{\partial^2 TC2}{\partial T^2} = \frac{1}{T}\left(D\left((h_o + c\alpha)e^{\alpha(T-t_w)} + (1-\delta)cI_p e^{\alpha(T-M)}\right) - \delta p D I_e - 2\frac{\partial TC2}{\partial T}\right) \tag{32}$$

Evaluating (32) at (t_w^{2*}, T_2^*) we obtain

$$\left.\frac{\partial^2 TC2}{\partial T^2}\right|_{(t_w^{2*},T_2^*)} = \frac{1}{T}\left(D\left((h_o + c\alpha)e^{\alpha(T-t_w)} + (1-\delta)cI_p e^{\alpha(T-M)}\right) - \delta p D I_e - 2\frac{\partial TC2}{\partial T}\right)\Big|_{(t_w^{2*},T_2^*)}$$

and since from equation (30), $\left.\frac{\partial TC2}{\partial T}\right|_{(t_w^{2*},T_2^*)} = 0$ we see that

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$\left.\frac{\partial^2 TC2}{\partial T^2}\right|_{(t_w^{2*},T_2^*)} = \frac{1}{T}\left(D\left((h_o + c\alpha)e^{\alpha(T-t_w)} + (1-\delta)cI_p e^{\alpha(T-M)}\right) - \delta pDI_e\right)\Big|_{(t_w^{2*},T_2^*)} > 0$if and only if $D\left((h_o + c\alpha)e^{\alpha(T-t_w)} + (1-\delta)cI_p e^{\alpha(T-M)}\right) - \delta pDI_e > 0$ which holds from the result of lemma 2 provided $(1-\delta)cI_p \geq \delta pDI_e$.

Using left hand side of equation (29), differentiating and simplifying, we see that

$$\frac{\partial^2 TC2}{\partial T \partial t_w} = -\frac{1}{T}\left(D(h_o + c\alpha)e^{\alpha(T-t_w)} + \frac{\partial TC2}{\partial t_w}\right) \tag{33}$$

Also, left hand side of using (30),

$$\frac{\partial^2 TC2}{\partial t_w \partial T} = -\frac{1}{T}\left(D(h_o + c\alpha)e^{\alpha(T-t_w)} + \frac{\partial TC2}{\partial t_w}\right) \tag{34}$$

Evaluating (33) and (34) at the point (t_w^{2*}, T_2^*) we see that

$$\left.\frac{\partial^2 TC2}{\partial T \partial t_w}\right|_{(t_w^{2*},T_2^*)} = -\frac{1}{T}\left(D(h_o + c\alpha)e^{\alpha(T-t_w)}\right)\Big|_{(t_w^{2*},T_2^*)} = \left.\frac{\partial^2 TC2}{\partial t_w \partial T}\right|_{(t_w^{2*},T_2^*)}$$

Theorem 2: if $(1-\delta)cI_p \geq \delta pI_e$ then TC2 in equation (16) is a convex function.

Proof

The proof follows from lemmas 1 and 2 and equations (31) – (34), since in that case $\left\{\frac{\partial^2 TC2}{\partial t_w^2}\frac{\partial^2 TC2}{\partial T^2} - \frac{\partial^2 TC2}{\partial t_w \partial T}\frac{\partial^2 TC2}{\partial T \partial t_w}\right\}\Big|_{(t_w^{2*},T_2^*)} > 0$ and $D > \alpha W$ from the assumption (e).

The necessary conditions for TC3 to have minimum are $\frac{\partial TC3}{\partial t_w} = 0$ and $\frac{\partial TC3}{\partial T} = 0$

Using (19), differentiating and setting the result to zero, we obtain

$$\frac{\partial TC3}{\partial t_w} = \frac{1}{T}\left(\frac{D}{\beta}(h_r + c\beta)\left(e^{\beta t_w} - 1\right) + W(h_o + c\alpha)e^{-\alpha t_w} + \frac{D}{\alpha}(h_o + c\alpha)\left(1 - e^{\alpha(T-t_w)}\right)\right) = 0 \tag{35}$$

Also, using equation (19) differentiating and setting the result to zero, we get

$$\frac{\partial TC3}{\partial T} = \frac{1}{T}\left(\frac{D}{\alpha}\left((h_o + c\alpha)\left(e^{\alpha(T-t_w)} - 1\right) + (1-\delta)(1-\gamma)cI_p\left(e^{\alpha(T-M)} - 1\right)\right) - pDI_e(M - (1-\gamma)N - T) - TC3\right) = 0 \tag{36}$$

The solutions to Equations (35) and (36) give the values t_w and T for the model in this case. To confirm that the solutions exist and are unique, we show that the determinant of the Hessian matrix evaluated at (t_w^{3*}, T_3^*) is positive definite.

Using left hand side of equation (35), we find that

$$\frac{\partial^2 TC3}{\partial t_w^2} = \frac{1}{T}\left(D(h_r + c\beta)e^{\beta t_w} - \alpha W(h_o + c\alpha)e^{-\alpha t_w} + D(h_o + c\alpha)e^{\alpha(T-t_w)}\right) \tag{37}$$

Evaluating (37) at (t_w^{3*}, T_3^*) we see that

$\left.\frac{\partial^2 TC3}{\partial t_w^2}\right|_{(t_w^{3*},T_3^*)} = \frac{1}{T}\left(D(h_r + c\beta)e^{\beta t_w} - \alpha W(h_o + c\alpha)e^{-\alpha t_w} + D(h_o + c\alpha)e^{\alpha(T-t_w)}\right)\Big|_{(t_w^{3*},T_3^*)} > 0$ If and only if $D(h_r + c\beta)e^{\beta t_w} - \alpha W(h_o + c\alpha)e^{-\alpha t_w} > 0$

Lemma 3: if $D > \alpha W$ then $D(h_r + c\beta)e^{\beta t} - \alpha W(h_o + \alpha c)e^{-\alpha t} > 0 \; for\ all\ t > 0$.

Proof

Let $g(t) = D(h_r + c\beta)e^{\beta t} - \alpha W(h_r + c\beta)e^{-\alpha t}$ for all $t > 0$

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$g(0) = (D - \alpha W)(h_r + c\beta) > 0$ since $D - \alpha W > 0$ from the assumptions.

$g'(t) = \beta D(h_r + c\beta)e^{\beta t} + \alpha^2 W(h_r + \alpha\beta)e^{-\alpha t} > 0$for all values of t.

Since $g'(t) > 0$, for all $t > 0$, we can conclude that g(t) is an increasing function of t

From the assumption (d), $h_r - h_o > c(\alpha - \beta) \Rightarrow h_r + c\beta > h_o + \alpha c$

$\Rightarrow D(h_r + c\beta)e^{\beta t} - \alpha W(h_o + \alpha c)e^{-\alpha t} > 0$ proved□

Using left hand side of equation (36), we see that

$$\frac{\partial^2 TC3}{\partial T^2} = \frac{1}{T}\left(D\left((h_o + c\alpha)e^{\alpha(T-t_w)} + (1-\delta)(1-\gamma)cI_p e^{\alpha(T-M)}\right) + pDI_e - 2\frac{\partial TC3}{\partial T}\right) \quad (38)$$

Evaluating (38) at (t_w^{3*}, T_3^*) we get

$$\left.\frac{\partial^2 TC3}{\partial T^2}\right|_{(t_w^{3*},T_3^*)} = \frac{1}{T}\left(D\left((h_o + c\alpha)e^{\alpha(T-t_w)} + (1-\delta)(1-\gamma)cI_p e^{\alpha(T-M)}\right) + pDI_e - 2\frac{\partial TC3}{\partial T}\right)\Bigg|_{(t_w^{3*},T_3^*)}$$

and since from equation (36), $\left.\frac{\partial TC3}{\partial T}\right|_{(t_w^{3*},T_3^*)} = 0$ we see that

$$\left.\frac{\partial^2 TC3}{\partial T^2}\right|_{(t_w^{3*},T_3^*)} = \frac{1}{T}\left(D\left((h_o + c\alpha)e^{\alpha(T-t_w)} + (1-\delta)(1-\gamma)cI_p e^{\alpha(T-M)}\right) + pDI_e\right)\Bigg|_{(t_w^{3*},T_3^*)} > 0$$

Using left hand side of equation (35),

$$\frac{\partial^2 TC3}{\partial T \partial t_w} = -\frac{1}{T}\left(D(h_o + c\alpha)e^{\alpha(T-t_w)} + \frac{\partial TC3}{\partial t_w}\right) \quad (39)$$

Also, using left hand side of (36),

$$\frac{\partial^2 TC3}{\partial t_w \partial T} = -\frac{1}{T}\left(D(h_o + c\alpha)e^{\alpha(T-t_w)} + \frac{\partial TC3}{\partial t_w}\right) \quad (40)$$

Evaluating (39) and (40) at the point (t_w^{3*}, T_3^*) we see that

$$\left.\frac{\partial^2 TC3}{\partial T \partial t_w}\right|_{(t_w^{3*},T_3^*)} = -\frac{1}{T}\left(D(h_o + c\alpha)e^{\alpha(T-t_w)}\right)\Big|_{(t_w^{3*},T_3^*)} = \left.\frac{\partial^2 TC3}{\partial t_w \partial T}\right|_{(t_w^{3*},T_3^*)}$$

Theorem 3: The cost function TC3 in (19) is a convex function.

<u>Proof</u>

The proof follows from lemma 3 and equations (37) – (40), since in that case $\left\{\frac{\partial^2 TC3}{\partial t_w^2}\frac{\partial^2 TC3}{\partial T^2} - \frac{\partial^2 TC3}{\partial t_w \partial T}\frac{\partial^2 TC3}{\partial T \partial t_w}\right\}\Bigg|_{(t_w^{3*},T_3^*)} > 0$

The necessary conditions for TC4 to have minimum are $\frac{\partial TC4}{\partial t_w} = 0$ and $\frac{\partial TC4}{\partial T} = 0$

Using (22), differentiating, and setting the result to zero, we get

$$\frac{\partial TC4}{\partial t_w} = \frac{1}{T}\left(\frac{D}{\beta}(h_r + c\beta)(e^{\beta t_w} - 1) + W(h_o + c\alpha)e^{-\alpha t_w} + \frac{D}{\alpha}(h_o + c\alpha)\left(1 - e^{\alpha(T-t_w)}\right)\right) = 0 \quad (41)$$

Again, using equation (22), differentiating and setting the result to zero, we obtain

$$\frac{\partial TC4}{\partial T} = \frac{1}{T}\left(\frac{D}{\alpha}\left((h_o + c\alpha)\left(e^{\alpha(T-t_w)} - 1\right)\right) - pDI_e(M - 2(1-\gamma)N - T) - TC4\right) = 0 \quad (42)$$

The solutions to Equations (41) and (42) give the values t_w and T for the model in this case. To confirm that the solutions exist and are unique, we show that the determinant of the Hessian matrix evaluated at (t_w^{4*}, T_4^*) is positive definite.

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Using equation (41), we see that

$$\frac{\partial^2 TC4}{\partial t_w^2} = \frac{1}{T}\left(D(h_r + c\beta)e^{\beta t_w} - \alpha W(h_o + c\alpha)e^{-\alpha t_w} + D(h_o + c\alpha)e^{\alpha(T-t_w)}\right) \quad (43)$$

Evaluating equation (43) at (t_w^{4*}, T_4^*) we see that

$\left.\frac{\partial^2 TC4}{\partial t_w^2}\right|_{(t_w^{4*},T_4^*)} = \frac{1}{T}\left(D(h_r + c\beta)e^{\beta t_w} - \alpha W(h_o + c\alpha)e^{-\alpha t_w} + D(h_o + c\alpha)e^{\alpha(T-t_w)}\right)\Big|_{(t_w^{4*},T_4^*)} > 0$ If and only if $D(h_r + c\beta)e^{\beta t_w} - \alpha W(h_o + c\alpha)e^{-\alpha t_w} > 0$ which holds from the result of lemma 3.

Using left hand side of equation (42),

$$\frac{\partial^2 TC4}{\partial T^2} = \frac{1}{T}\left(D(h_o + c\alpha)e^{\alpha(T-t_w)} + pDI_e - 2\frac{\partial TC4}{\partial T}\right) \quad (44)$$

Evaluating (44) at (t_w^{4*}, T_4^*) we get

$$\left.\frac{\partial^2 TC4}{\partial T^2}\right|_{(t_w^{4*},T_4^*)} = \frac{1}{T}\left(D(h_o + c\alpha)e^{\alpha(T-t_w)} + pDI_e - 2\frac{\partial TC4}{\partial T}\right)\Big|_{(t_w^{4*},T_4^*)}$$

and since from equation (42), $\left.\frac{\partial TC4}{\partial T}\right|_{(t_w^{4*},T_4^*)} = 0$ we see that

$$\left.\frac{\partial^2 TC4}{\partial T^2}\right|_{(t_w^{4*},T_4^*)} = \frac{1}{T}\left(D(h_o + c\alpha)e^{\alpha(T-t_w)} + pDI_e\right)\Big|_{(t_w^{4*},T_4^*)} > 0$$

Using left hand side of equation (41),

$$\frac{\partial^2 TC4}{\partial T \partial t_w} = -\frac{1}{T}\left(D(h_o + c\alpha)e^{\alpha(T-t_w)} + \frac{\partial TC4}{\partial t_w}\right) \quad (45)$$

Also, using left hand side of (42), we have

$$\frac{\partial^2 TC4}{\partial t_w \partial T} = -\frac{1}{T}\left(D(h_o + c\alpha)e^{\alpha(T-t_w)} + \frac{\partial TC4}{\partial t_w}\right) \quad (46)$$

Evaluating (45) and (46) at the point (t_w^{4*}, T_4^*) we see that

$$\left.\frac{\partial^2 TC4}{\partial T \partial t_w}\right|_{(t_w^{4*},T_4^*)} = -\frac{1}{T}\left(D(h_o + c\alpha)e^{\alpha(T-t_w)}\right)\Big|_{(t_w^{4*},T_4^*)} = \left.\frac{\partial^2 TC4}{\partial t_w \partial T}\right|_{(t_w^{4*},T_4^*)}$$

Theorem 4: The cost function TC4 in (22) is a convex function.

Proof

The proof follows from equations (43) – (46) since $\left\{\frac{\partial^2 TC4}{\partial t_w^2}\frac{\partial^2 TC4}{\partial T^2} - \frac{\partial^2 TC4}{\partial t_w \partial T}\frac{\partial^2 TC4}{\partial T \partial t_w}\right\}\Big|_{(t_w^{4*},T_4^*)} > 0$

6. **Numerical Example**

Suppose we consider the situation with the following inventory parameters as in Liang and Zhou (2011):

A = 1500, D = 2000, W = 100, c = 10, p = 15, h_r = 3, h_o = 1, β = 0.06, α = 0.1, γ = 0.6, δ = 0.4M = 0.5, N = 0.25, I_e = 0.12, I_p = 0.15.

Putting in thevalues of the parameters abovein equations (7), (13), (16), (19) and (22), we obtain the following results presented in Table 1 below:

Table 1: Empirical Solution

Cases	t_w	T	TC

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Case 1	0.5671	0.6143	3842.70
Case 2	0.5425	0.5897	3994.71
Case 3	0.3973	0.4452	3954.45
Case 4	0.4738	0.5215	4488.12

Since the cost N3824.70 for case 1 is the least, we select that the case as our best solution. Therefore, t_w^* = 0.5671 (207 $days$), T^* = 0.6143 (224 $days$) and $\boldsymbol{TC}^*$ = 3842.700

Sensitivity Analysis: we now study the effect of parameter changes (Sensitivity analysis) of the inventory parameters W, A and D on the optimal policies of the model (using the example above). The new parameters used are $W \in$ (100, 250, 400), $A \in$ (1500, 2000, 2500) and $D \in$ (2000, 2500, 3000).

Table 2: Sensitivity analysis on the example above.

W	A	D	t_w	T	TC
		2000	0.5671	0.6143	3842.700
	1500	2500	0.5123	0.5503	4195.304
		3000	0.4767	0.5084	4493.246
		2000	0.6630	0.7097	4597.186
100	2000	2500	0.5973	0.6349	5038.642
		3000	0.5479	0.5795	5416.825
		2000	0.7452	0.7915	5262.650
	2500	2500	0.6712	0.7086	5782.007
		3000	0.6164	0.6477	6230.871
		2000	0.5014	0.6196	3644.872
	1500	2500	0.4575	0.5526	3993.940
		3000	0.4247	0.5042	4288.855
		2000	0.5945	0.7116	4396.244
250	2000	2500	0.5425	0.6367	4834.306
		3000	0.5014	0.5803	5210.029

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	2500	2000	0.6795	0.7956	5060.030
		2500	0.6164	0.7100	5576.052
		3000	0.5699	0.6483	6022.490
400	1500	2000	0.4356	0.6253	3472.437
		2500	0.4055	0.5580	3815.676
		3000	0.3808	0.5084	4106.301
	2000	2000	0.5288	0.7167	4216.830
		2500	0.4904	0.6416	4649.575
		3000	0.4575	0.5841	5021.362
	2500	2000	0.6137	0.8000	4876.263
		2500	0.5644	0.7145	5387.292
		3000	0.5260	0.6517	5830.031

From the table 2 above, we can deduce the following:

a. The retailer incurs highest total relevant cost (*TC=6230.871*) when the capacity of own warehouse is small, *W=100*, and the ordering cost, *A=2500* and the demand, *D=3000*. This is expected as the smaller the capacity (precisely, *W=100*) of retailer's own warehouse the higher the holding cost incurred by the retailer and this leads to increase in *TC*.
b. The retailer incurs lowest total relevant cost (*TC=3472.437*) when the stocking capacity of own warehouse is increased, *W=400*, the ordering cost remain unchanged from the initial value, *A=1500*, and the demand increases, *D=2000*. This is also expected since the retailer incurs lower holding cost in *OW* thereby lowering the total relevant cost.

6**. Conclusion**

In this paper, we have developed an EOQ model for the two – warehouse inventory system and looked at situations when both the retailer and retailer's customers are considered as credit – risk. This is as a result of negative effect of failure in payment on a business transaction that involves trade credit financing (two – level). We have incorporated the partial upstream to check credit riskiness of the retailer and partial downstream to check the credit riskiness of the customers. There are four possible scenarios for the model and convexity has been established for all the scenarios. A numerical example has been given and solved to see how the different scenarios perform and which of the scenarios gives the best result. A sensitivity analysis has then been carried out on the example to see the changes in system parameters.

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